§4. High-beta Axisymmetric Equilibria with Flow in Reduced Single-fluid and Two-fluid Models

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In improved confinement modes of magnetically confined plasmas where high-beta is achieved, equilibrium flows play important roles like the suppression of instability and turbulent transport. At the sharp boundary of a well-confined region, the scale lengths characteristic of microscopic effects not included in single-fluid magnetohydrodynamics (MHD) cannot be neglected. Small scale effects on flowing equilibria due to the Hall current have been studied with two-fluid or Hall MHD models. However, these models are consistent with kinetic theory only for cold ions [1]. In order to include the hot ion effects that are relevant to fusion plasmas, an extension of the model is necessary. A consistent treatment of hot ions in a two-fluid framework must include the ion gyroviscosity and other finite Larmor radius (FLR) effects. In the fluid formalism of collisionless magnetized plasmas, these effects are incorporated by means of asymptotic expansions in terms of the small parameter $\delta \sim \rho_i/a$, where ρ_i is the ion Larmor radius and a is the macroscopic scale length. With a slow dynamics ordering, $v \sim \delta v_{th}$ where v and v_{th} are the flow and thermal velocities respectively, the ion FLR terms [2,3] are much simplified in the reduced models for large-aspect-ratio, high-beta tokamaks [4,5] after relating δ to the inverse aspect ratio expansion parameter $\varepsilon \equiv a/R_0 \ll 1$, where a and R_0 are the characteristic scale lengths of the minor and major radii respectively [4,5].

We have derived the equations for high-beta axisymmetric equilibria with flow comparable to the poloidal Alfvén velocity in the reduced two-fluid model with FLR and flow comparable to the poloidal sound velocity in the single-fluid model, by using asymptotic expansions in terms of the inverse aspect ratio [6]. These velocities are the characteristic velocities that bring singularities in the equilibrium equations. The poloidal-Alfvenic flow is of interest because the equations for axisymmetric equilibria in single-fluid MHD have a singularity when the poloidal flow velocity is equal to the poloidal Alfven velocity, the so-called Alfven singularity. This can be described by the reduced model with the relation $\delta^2 \sim \epsilon$. The poloidal-sonic flow is of interest because the equilibria show a discontinuity at the point where the poloidal flow velocity crosses the poloidal sound velocity. This can be described by the reduced model with the relation $\delta \sim \epsilon$. While the poloidal-Alfvenic flow analysis follows the standard orderings of reduced MHD for high-beta tokamaks, the poloidal-sonic flow analysis does not and higher-order terms must be taken into account. Since the formulation of higher-order equations is involved, here we restrict our analysis of the poloidal-sonic flow to the single-fluid case, planning to extend our present results with the inclusion of two-fluid, hot ion effects in future work. The orderings in this study provide the simplest models that include ion FLR effects on toroidal equilibria with flow. As such, they should be just considered as convenient working hypotheses that allow our analytic study of such effects.

We have shown that the Alfvén singularity is shifted by the gyroviscous cancellation. The singularity at the poloidal flow velocity equal to the poloidal sound velocity in the density and pressure and its dependence on toroidicity have been reproduced by our higher-order terms and the singularity in the higher-order magnetic structure has been found. The reduced single-fluid equations for equilibria with poloidal-sonic flow include higher-order quantities and hence can describe finite-aspect-ratio tokamak equilibria. The resulting equations can be easily solved numerically to yield flowing equilibria without singularity and their solutions can be used as initial states or for comparison with saturated states of reduced model nonlinear simulations.

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