§27. Research Plan of Cauchy-condition Surface Method Analysis to Reconstruct 3-D Plasma Boundary Profile

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1. Introduction

The boundary shape of fusion plasma can provide information related to the MHD equilibrium. Also, on-line computing of the shape is important from a viewpoint of operating control. For a tokamak device, the boundary shape analysis based on the Cauchy-condition surface (CCS) method[1)2)] with the magnetic sensor signals has been successfully made in a two-dimensional (2-D) system, since the tokamak plasma is regarded as axisymmetric in the toroidal direction. Solid lines in Fig.1 show the contours of the magnetic flux obtained using the 2-D CCS method. The outermost closed flux surface agrees well with the bold dotted line that denotes the true plasma boundary in JT-60.

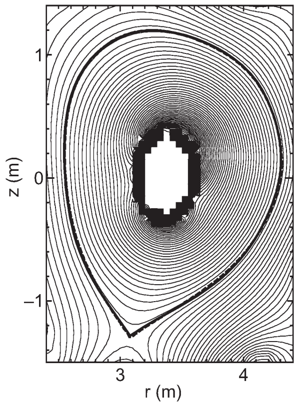

Fig.1 Reconstructed plasma boundary for JT-60

A 3-D version of the CCS method must be developed if one attempts to analyze the boundary profile of plasma that has no axisymmetry, e.g., a helical type device like LHD. Unfortunately it has not been well investigated whether such a 3-D inverse problem has a unique solution.

2. Inverse analysis using 3-D CCS method

Figure 2 illustrates the image of 3-D CCS method. One locates a CCS, a surface where both the Dirichlet and the Neumann conditions are unknown, inside the actual plasma region.

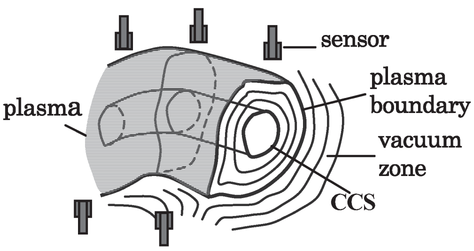

Fig.2 Image of 3-D CCS method

In this 3-D case, the number of unknowns distributed on the tube-shape CCS tends to be over one hundred. As a large number of unknowns leads to a poor results of inverse analysis, this is a challenging problem.

3. Formulation of 3-D CCS method

Suppose a 3-D plasma domain, which has a current density distribution $\mathbf{J}$, is surrounded by an infinite vacuum zone. The vector potential $\mathbf{A}=(A_1, A_2, A_3)$ in the 3-D space satisfies the 3-D Poisson equation

$$\nabla^2\mathbf{A} = -\mu_0\mathbf{J}\,. \quad (1)$$

However, in the inverse analysis to determine the boundary shape, one assumes a vacuum field everywhere outside the CCS even in the actual plasma domain. Three types of boundary integral equations for the vacuum field are given as follows.

For magnetic sensor positions ‘i’:

$$\int_{\Gamma_{CCS}}\left(\psi^*\frac{\partial A_j}{\partial n} - A_j\frac{\partial \psi^*}{\partial n}\right)d\Gamma = A_j^{(i)} - W_j^{(i)} \quad (j=1,2,3)\,, \quad (2)$$

and

$$\int_{\Gamma_{CCS}}\left(\frac{\partial \psi^*}{\partial m}\frac{\partial A_j}{\partial n} - A_j\frac{\partial^2 \psi^*}{\partial m\partial n}\right)d\Gamma = \frac{\partial A_j^{(i)}}{\partial m} - \frac{\partial}{\partial m}W_j^{(i)} \quad (j=1,2,3)\,. \quad (3)$$

For points ’i’ along the CCS:

$$\int_{\Gamma_{CCS}}\left(\psi^*\frac{\partial A_j}{\partial n} - A_j\frac{\partial \psi^*}{\partial n}\right)d\Gamma = \frac{1}{2}A_j^{(i)} - W_j^{(i)} \quad (j=1,2,3)\,. \quad (4)$$

Here, the quantity ψ^* is the fundamental solution that satisfies

$$\nabla^2\psi^* + \delta_i = 0\,, \quad (5)$$

while $W_j^{(i)}$ denotes the contribution of all coil currents to the point ‘i’. Equations (2), (3) and (4) are discretized, coupled and solved.

Once all the values of $\partial A_j/\partial n$ and A_j along the CCS (Γ_{CCS}) have been given, the distribution of vector potential can be calculated using the formula

$$A_j^{(i)} = \int_{\Gamma_{CCS}}\left(\psi^*\frac{\partial A_j}{\partial n} - A_j\frac{\partial \psi^*}{\partial n}\right)d\Gamma + W_j^{(i)} \quad (j=1,2,3) \quad (6)$$

for arbitrary points ‘i’. The outermost closed magnetic flux surface, i.e., the plasma boundary, can be found by drawing contours of magnetic flux following Eq.(6).

4. Research plan

The authors plan to expand the application of the CCS method to the analyses of 3-D plasmas. For this purpose, a new 3-D boundary element computer code based on ‘discontinuous quadratic elements’ is now under development. Benchmark analyses and feasibility study on the application in an actual fusion device are also scheduled.

1) Kurihara, K., *Fusion Eng. Des.*, **51-52** 1049 (2000).
2) Itagaki, M. et al., *Nuclear Fusion*, **45**, 153- 162 (2005).