§21. Reconstruction of the Eddy Current Profile on the Vacuum Vessel in a Reversed Field Pinch Device Using only External Magnetic Sensor Signals

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1. Introduction

For the MHD equilibrium reconstruction of a reversed field pinch device, it is a big issue to identify the strong eddy current flow on the shell (vacuum vessel). In the present work, boundary integrals of the eddy current along the shell are added to the conventional Cauchy-condition surface method formulation. The eddy current profile is unknown in advance but straightforwardly identified using only the signals from magnetic sensors located outside the plasma. The capability of the method is demonstrated for the RELAX device, a reversed field pinch device[1].

2. Method

The following two ideas are introduced to overcome the numerical difficulties encountered in the problem.

2.1 Accurate boundary integrals along the shell

As the sensors are closely adjacent to the shell, the near singular boundary integrals along the shell should be accurately evaluated. The general form of a boundary integral over a shell can be arranged as

$$\int_{-1}^{1} \phi(\xi)G(\xi)B^{*}(\xi)d\xi$$
$$= \int_{-1}^{1}\left\{\phi(\xi)G(\xi)B^{*}(\xi)-\phi_0 G_0 F_S(\xi)\right\}d\xi+\phi_0 G_0\int_{-1}^{1}F_S(\xi)d\xi ,$$

where G_0 and ϕ_0 are the values of Jacobian $G(\xi)$ and interpolation function $\phi(\xi)$ at the position $\xi=\xi_0$ on the boundary element that is the nearest to the location of the sensor under consideration.

The asymptotic function $F_S(\xi)$ is subtracted from the original integrand, and this subtraction is compensated by the analytical integral of the 2nd term on the RHS. The total of the 1st integral has no singularity and can therefore be evaluated with the ordinary Gaussian quadrature.

2.2 Solving an ill-conditioned matrix equation

The set of boundary integral equations is converted into a matrix equation, and the matrix $\mathbf{D}$ is decomposed as $\mathbf{D}=\mathbf{U}\mathbf{\Lambda}\mathbf{V}^{T}$, where $\mathbf{\Lambda}$ is a diagonal matrix with non-negative singular values (SVs). Figure 1 shows the behavior of the SVs for various numbers of assumed eddy current nodes. A gap is commonly observed in the vicinity of 10^{-3} of the SVs. To obtain a solution stable against sensor signal noise, it is important to adopt enough number of eddy current nodes to ensure that all SVs larger than the "gap threshold" are taken into account. That is, the best choice is the use of 40 current nodes in this case.

A numerical oscillation of the solution is observed if the smallest SV is smaller than the gap threshold, i.e., if the number of current nodes is larger than 40. To suppress the oscillation, one introduces the modified truncated SV decomposition (MTSVD) technique[2], which is based on a constraint, $\min\|\mathbf{Lp}\|$ with $\mathbf{L}$ being a differential operator.

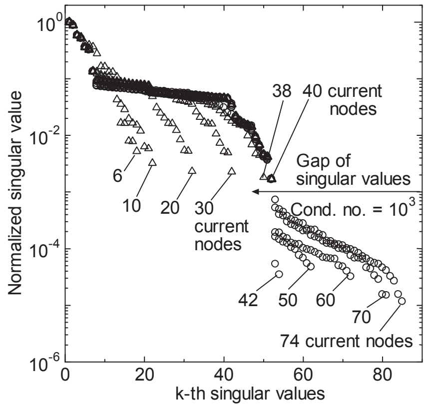

Fig.1 Singular values with the number of eddy current nodes

3. Numerical demonstration

The present techniques enable one to identify accurately not only the magnetic flux profile outside the plasma but the eddy current distribution itself. Figure 2 shows the variation in the eddy current density on the shell surface for the case assuming 60 eddy current nodes. The abscissa means the poloidal angle that varies in the clockwise direction on the shell.

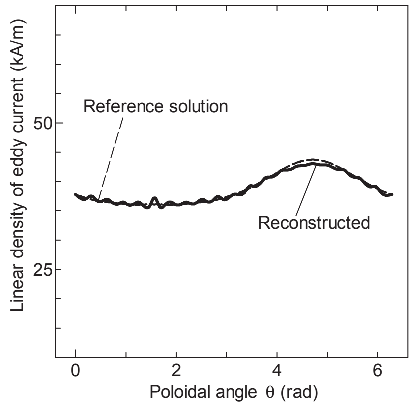

Fig.2 Reconstruction of the eddy current profile.

4. Conclusion

The singularity encountered on a boundary integral along the shell is damped out effectively with the algorithm based on the subtraction method. The numerical oscillation of the eddy current observed when using a large number of nodes can be eliminated effectively by applying the MTSVD technique.

The authors believe that the present techniques are applicable to the problem of eddy current flow in a conductor located close to a sensor in many other devices.

1) Itagaki, M., Sanpei, A., Masamune, S. & Watanabe, K., *Plasma Fusion Res.*, **9**, 1402046 (2014).
2) Hansen, P.C., Sekii, T. & Shibahashi, H., *SIAM J. Sci. Stat. Comput.*, **13**, 1142, 1992.