§2. Influence of Numerical Integration on Convergence of Eigenvalues in the MHD Stability Analysis

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Most of the MHD spectrum analyzing codes utilize numerical integrations, e.g., Gaussian Quadrature formulas to evaluate the energy integral. There are several reasons for this. One may be that coefficients in the integration of the energy integrals are not constant in most cases. Another reason may be that the physical quantities derived by equilibrium codes are numerically given in the actual stability calculations. The use of such a quadrature is a kind of variational crimes; most of theories of finite element method depend on an assumption that energy integrals are carried out exactly.

The use of quadrature formulas brings errors into the approximated eigenvalue in addition to primary errors by the finite element method. When the errors caused by the quadrature formulas are much smaller than the primary ones by the finite element method, the effect of numerical integrations are not substantial; the lowest eigenvalue is approximated from above,i.e., the approximated eigenvalue decreases towards the true eigenvalue as the number of elements increases. When the errors by the quadrature formulas are of the same order of the errors by the finite element method, the convergence properties of eigenvalues would be much affected, and the lowest eigenvalue is not always approximated from above. In most calculations by ERATO code or CAS3D code, the lowest eigenvalue is approximated from below, i.e., the approximated eigenvalue increases towards the true eigenvalue as the number of elements increases. This may be explained by the feature of the Gaussian quadrature formulas used by these codes.

In the present study, we investigate the Gaussian quadrature formulas, of which errors are of the order of the primary errors by the finite element method, to the convergence property of the lowest eigenvalue.

We consider torus plasma configurations. The magnetic surfaces are labeled by ψ, $\psi = 0$ being the magnetic axis, and $\psi = 1$ plasma surface. As an example, we consider the case that profile functions are $p(r) = 0.0015(1 - r^2)^2$ and $q(r) = 1.45 + 0.2r^2$. When we choose the poloidal mode number m=3 and n=-2, the interchange mode becomes unstable. In Fig.1, the calculated eigenvalues λ_h^* and the values of errors μ_h (definitions of λ_h^* and μ_h are given in Ref.1) are plotted with the fitting line versus N^{-2}. Both the calculated eigenvalues λ_h^* and the errors μ_h converge from below, respectively. In this case, the slopes of two fitting lines agree fairly well. The relative errors is about 30%.

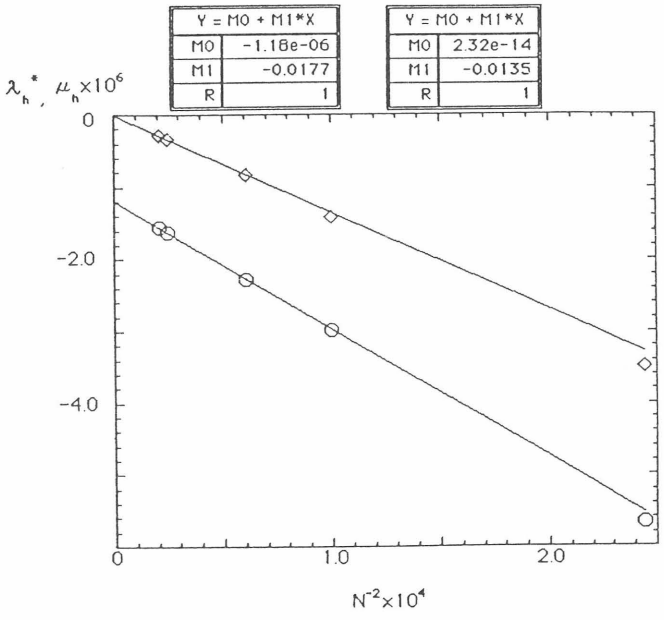

Fig.1 Approximated eigenvalues λ_h^* and errors μ_h plotted as a function of the number of intervals N^{-2} in case of the interchange mode.