## §39. An Extended K-dV Equation for Nonlinear Magnetosonic Wave in a Multi-Ion Plasma

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Much attention has been paid recently on the nonlinear magnetosonic waves because on the one hand they play an important role on particle acceleration and heating of plasmas, on the other hand its nonlinear behaviour is an attractive subject from viewpoint of nonlinear wave phenomena. For a plasma with one ion species, the linear dispersion relaton of a magnetosonic wave propagating perp-pendicularly to a magnetic field is given as

$$\omega \cong kV_A - ak^3.$$

It is well known that the nonlinear evolution of a magnetosonic wave in a plasma with one -ion species is described by the so called K-dV equation. For two ion plasma, however, the dispersion branch is split into two modes, namely, the high and low frequency modes. Since the dispersion branch of this high frequency mode has a finite cut-off frequency and depends sensitively on the ordering for smallness parameter $m_e/m_i$ , we have to pay attention to the ordering to discuss accurately the influence of finite cut-off frequency on the high frequency nonlinear magnetosonic wave. In the previous discussion, K-dV equation has been derived based on the assumption, $m_e/m_i \sim \varepsilon^3$. The linear dispersion relation for high frequncy magnetosonic wave under the influence of the finite cut-off frequency in the plasma with two ion-species is described by

$$\omega \cong kV_A - ak^3 + \frac{b}{k}$$

provided $m_e/m_i \sim \varepsilon^2$ is applied. Here, the coefficients a and b are given as

$$a = \frac{3}{2}\frac{\omega_{pe}^2(\omega_{pa}^2\Omega_a + \omega_{pb}^2\Omega_b)}{\Omega_e(\omega_{pa}^2 + \omega_{pb}^2)^2}$$

$$b = \frac{V}{2c^2}\frac{\omega_{pa}^2\omega_{pb}^2(\Omega_a - \Omega_b)}{(\omega_{pa}^2 + \omega_{pb}^2)}\left\{\frac{\omega_{pa}^2}{\Omega_a^2} + \frac{\omega_{pb}^2}{\Omega_b^2}\right\},$$

and we note that the coefficient b tends to disappear in case of single ion plasma.

As was discussed in the last year's report, the nonlinear evolution of this mode is described by the following an extended K-dV equation

$$\frac{\partial}{\partial \xi}\left\{\frac{\partial u}{\partial \tau} + \beta u \frac{\partial u}{\partial \xi} + \gamma \frac{\partial^3 u}{\partial \xi^3}\right\} - \delta u = 0$$

where $\beta$ =a , $\delta$=b and $\gamma = \frac{c^2 V}{2\omega_{pe}^2}$.

Also, the boundary condition

$\int_{-\infty}^{\infty} u d\xi \approx 0$ is obtained. But, this condition is different from that for conventional K-dV equation. It should be noted that our nonlinear equation seems to have only three converved quantities although K-dV equation has infinite number of converved ones. General characteristics of the solution of our nonlinear equation are now numerically studied.

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