§1. CLOTHO: MHD Stability Code with Higher Order Spline Functions

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The new code for the analysis of the ideal MHD instability for 2-D equilibria is constructed. The code (CLOTHO code) achieves the fourth or higher order accuracy by using the finite element method with higher order elements. The outline of the CLOTHO code is as follows[1].

Suppose the torus plasma bounded by a conducting shell. The coordinate system (ψ, θ, ζ) is introduced where ζ accords with the toroidal angle ϕ which appears in the geometric cylindrical coordinate (R, ϕ, Z), and θ is determined by the condition that the lines of force are straight in the $\theta - \zeta$ plane , while ψ is taken as poloidal flux.

Equilibria are calculated by using the H-APOLLO code. The interface routine of the H-ERATO code is used to compute the equilibrium quantities at any point in (ψ, θ)-mesh from $\psi(R, Z)$.

The two parameters p and N are key parameters in the CLOTHO code. The parameter N determines the number of elements. When the plasma displacement vector is decomposed into the directions normal to the magnetic surface and in the magnetic surface, the components normal to the magnetic surface are expanded with B-spline functions of degree p and the components in the magnetic surface with B-spline functions of degree $p-1$(p = 1,2,3). These base functions are selected to satisfy the mathematical conditions which are given by Descloux-Nassif-Rappaz[2] to ensure the efficiency of approximations for the spectrum of a non-compact operator including the ideal linearized MHD operator. The integration in the energy integrals is numerically carried out by using p-point Gaussian quadrature formula over each element, and the eigenvalue problem for the matrices is derived. The eigenvalues are calculated by the Lanczos algorithm and the bisection method. The inverse iteration method is used to obtain the eigenvectors. The eigenvalues for unstable modes are approximated from bellow (see Fig.2) and the numeical errors in the eigenvalues scale as N^{-2p}.

An example is here studied. The equilibrium is calculated with the profiles $p(\psi) = 0.05(1 - \psi^2), q(\psi) = 0.6(1 + 2.33\psi^{3/2})$. The average beta is $<\beta>= 2.865\%$ and the aspect ratio is $R/a = 5$. In this equilibrium the internal kink mode becomes unstable. The B-spline functions of second degree and piecewise linear functions are used in hybrid as the finite element basis functions (p=2). In Fig.1 the plasma flow pattern at $\zeta = 0$ is illustrated in case of the number of elements $N = 20$. In Fig.2 the symbols $\circ$ represent the calculated eigenvalues in cases of $N = 16, 18, 20, 24, 28, 32$. These values are plotted with the fitting line versus N^{-4}. Even with $N = 20$, the numeical errors in the calculated eigenvalue scale as N^{-4}.

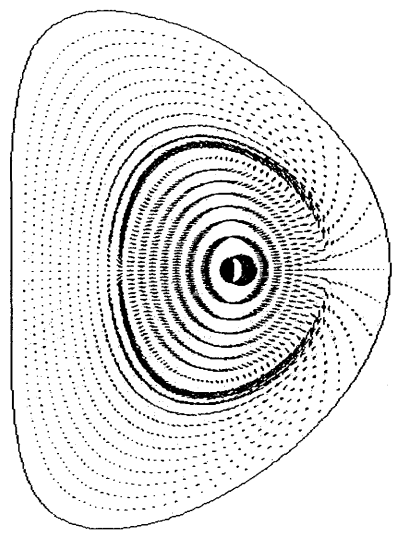

Figure 1: The plasma flow pattern at $\zeta = 0$ for the internal kink mode in case of $N = 20$.

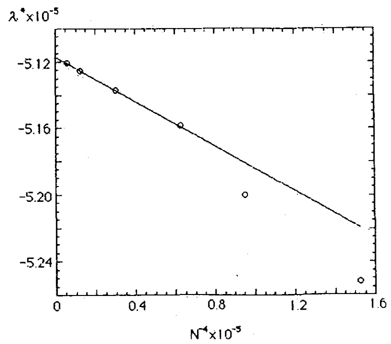

Figure 2: The calculated eigenvalues as function of N^{-4}

References

1) A.Ida, J.Todoroki, H.Sanuki, J.Plasma and Fusion Res. 75, May(1999) to be published
2) J.Descloux, N.Nassif, J.Rappaz, RAIRO Anal. Numér. 12 (1978) 97