§9. Resonant Property of Parallel Current in 3D MHD Equilibrium

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Existence of 3D MHD equilibrium with nested flux surfaces has not proved mathematically, and it seems to be negative i.e. the magnetic island or the stochastic region may be necessary in any 3D MHD equilibrium. If we use the covariant components of the magnetic field in the Boozer coordinates, the parallel current of a currentless equilibrium can be expressed by

$$\mathbf{J}\cdot\mathbf{B} = \frac{P'}{\sqrt{g_B}} \sum_{m\neq0\,\mathrm{or}\,n\neq0} \frac{mJ}{m\iota - n}\sqrt{g_B}_{mn}\cos(m\theta-n\zeta),$$

(1)

where J denotes the total poloidal current flowing outside the magnetic flux and $\sqrt{g_B}$ is the Jacobian. This equation implies that the parallel current is resonant at any rational surface unless $P' = 0$ or $\sqrt{g_B}_{mn} = 0$ is satisfied at the surface. The resonant current brings the magnetic island or the stochastic region. The method of (1) is called indirect method. Here we investigated the resonance in the parallel current with the VMEC code . In the VMEC code, the 3D equilibrium is calculated approximately under the assumption of the nested surfaces. However, the code does not deal with the resonant effect directly, because the parallel current is calculated with the contravarinat components of the magnetic field in the VMEC coordinates which is given by

$$\mathbf{J}\cdot\mathbf{B} = \sqrt{g_V}\epsilon_{ijk}B^i\partial_j B^k.$$

(2)

This method of (2) is called direct method. In order to know how the resonant property is, we estimated the parallel current with both methods through the Mercier criterion by using the results of the VMEC code. Here we use the LHD equilibria under the free boundary condition with the separatrix constraint. Below $\beta_0 = 5\%$, where β_0 denotes the beta value at the magnetic axis, the results with

the direct and the indirect methods agree with each other very well. At $\beta_0 = 6\%$, as shown in Fig.1, isolated spikes appear in the result of the indirect method which reflects the resonance of the parallel current, and therefore the existence of the islands. As beta value increases, the number of spikes increases and the difference between the two methods becomes large which shows the growth of the islands. At $\beta_0 = 8\%$, the spikes overlap each other in the large region and the indirect method indicates the completely different results from that with the direct method. The overlapping of the spikes may indicate the existence of the stochastic region due to the overlapping islands. In this sense, the large difference between the two methods may give the beta limit of approximate solution with nested surfaces.
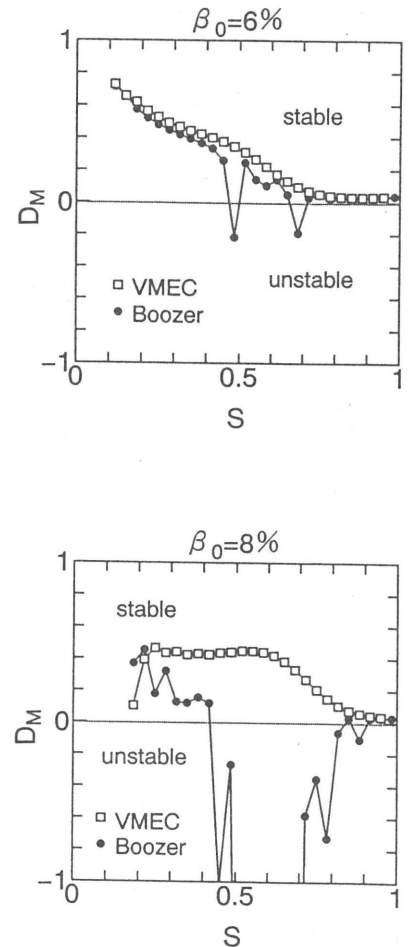

Fig.1 Results of Mercier criterion estimated with direct (open squares) and indirect (closed squares) methods.