§5. Extraction of Hierarchical Energy Spectrum in Turbulence and its Correlation with Organized Structure

Horiuti, K. (Tokyo Institute of Technology),
Miura, H.,
Tsubokura, M. (Hokkaido Univ.)

The structure of the energy spectrum in the inertial subrange is studied using the DNS data for turbulence in a periodic box at higher Reynolds numbers. The Kolmogorov -5/3 spectrum forms a base state for the energy spectrum in the inertial subrange. However, the spectrum which exhibits a scaling different from the -5/3 power is often depicted in turbulent flows. In fact, perturbation expansion about the Kolmogorov spectrum yields a correction with the -7/3 power. When the expansion is extended to higher orders, we obtain the formula as

$$E(k) = C_K \varepsilon^{2/3} k^{-5/3} + C_{7/3} \dot{\varepsilon} \varepsilon^{-2/3} k^{-7/3} + C_{9/3} \ddot{\varepsilon} \varepsilon^{-1} k^{-9/3} \quad (1)$$

which are induced by the fluctuation of the dissipation rate ε and represents a nonequilibrium state. $\dot{\varepsilon}$ denotes the time derivative of ε. We applied the averaging to the ensemble of the energy spectra conditioned on temporal variation of ε.

Assessment is carried out using the DNS data for forced homogeneous isotropic turbulence which are in statistically steady state. It is shown that the entire temporal development is divided into the three phases. In Phase 1, $\dot{\varepsilon} > 0$ and in Phase 2, $\dot{\varepsilon} < 0$, and in the transient period between Phases 1 and 2, $\dot{\varepsilon} \approx 0$ but $\ddot{\varepsilon}$ is large (Phase T). Figure 1 shows the energy spectrum averaged in the time interval $t_m - 10\,\tau < t < t_m + 10\,\tau$, where t_m is the time when ε is minimal or maximal, and τ is the Kolmogorov time scale. The dashed line shows the whole average of the spectrum $E_0(k)$, and the solid line shows the absolute values of deviation of the spectra from $E_0(k)$ averaged in Phases 1 and 2, $E_1(k)$. The absolute values of deviation of the spectra from $E_0(k)$ averaged in Phase T, $E_2(k)$, is plotted using the dots. As is indicated in Eq.(1), the base steady state fits the -5/3 power, but $E_1(k)$ and $E_2(k)$ exhibit fitting with the -7/3 and -9/3 power, respectively. In Phase T, the magnitude of the -7/3 component becomes small, and the -9/3 component predominates.

In Kaneda *et al.* (2003), it was shown that the slope in the inertial subrange is steeper than -5/3 by $\mu \sim 0.1$ at $R_\lambda \sim 1200$. Similar result was obtained in Tsuji (2004). We investigated on this deviation using the data from Run 4096-1 in Kaneda *et al.* (2003). The long-term average of (1) yields $E(k) = C_K \varepsilon^{2/3} k^{-5/3} + C_2 \dot{\varepsilon}^2 \varepsilon^{-2} k^{-9/3}$. When the spectrum is fitted using the function $E(k) = C_K \varepsilon^{2/3} k^{-(5/3+\mu)}$ by the least square method, $\mu \sim 0.083$ and $C_K \sim 2.24$. The fitting is done using the data in the range $0.00369 < k\eta < 0.0132$. μ is close to that in Kaneda *et al.* (2003), whereas C_K is larger than the generally accepted values (C_K =1.6-1.7). When the spectrum is fitted using the function $E(k) = C_K \varepsilon^{2/3} k^{-5/3} + C_3 k^{-9/3}$, $C_K \sim 1.688$ and $C_3 \sim 0.557$. (Because the value of $\dot{\varepsilon}$ was not available, the coefficient of $k^{-9/3}$ term, C_3, is optimised as a whole.) We note that when the exponent of the second term is determined using the function $E(k) = C_K \varepsilon^{2/3} k^{-5/3} + C_3 k^{-\alpha}$, $\alpha \sim 2.92$ ($C_K \sim 1.675$, $C_3 \sim 0.5045$). α is indeed close to 3. Figure 2 shows the energy spectra, $E(k) = C_K \varepsilon^{2/3} k^{-5/3} + C_3 k^{-9/3}$, $E(k) = C_1 k^{-7/3}$, $E(k) = C_3 k^{-9/3}$, where C_K =1.688, C_1=0.224 and C_3=0.557. It is seen that the deviation of the spectrum from $E_0(k)$ approximately fits the function $C_3 k^{-9/3}$ in the region $0.00369 < k\eta < 0.0132$ (indicated by the bald solid line and the arrows). It is shown that the deviation from -5/3 power can be approximated using the -9/3 term. These results indicate that the -9/3 component plays important roles as well as the -7/3 component at high R_λ.

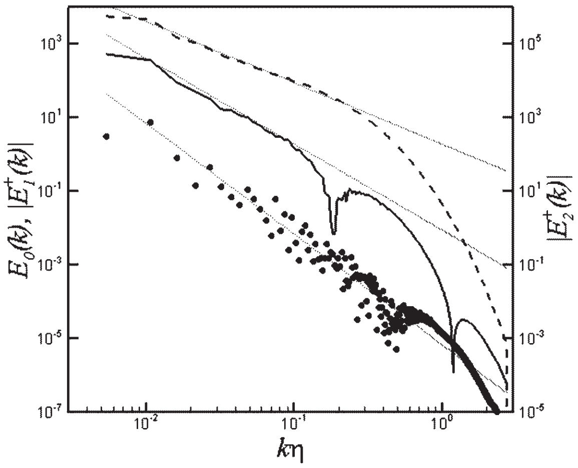

Fig. 1 Normalised energy spectra at $R_\lambda \sim 240$ as functions of $k\eta$. $E_0(k)$ is plotted using dashed line, $|E_1(k)|$ using solid line and $|E_2(k)|$ using dots. The dotted lines indicate scaling with $k^{-5/3}$, $k^{-7/3}$ and $k^{-9/3}$.

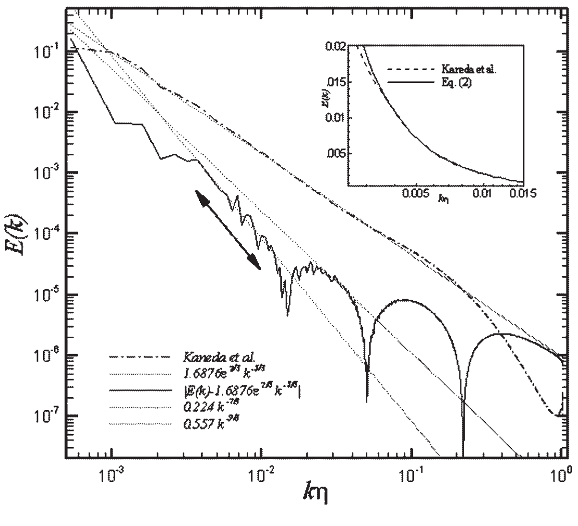

Fig. 2 Normalised energy spectra at $R_\lambda \sim 1200$ as functions of $k\eta$. $E_0(k)$ is shown using dashed-dotted line, $|E(k)-E_0(k)|$ using solid line. The dotted lines indicate scaling with $k^{-7/3}$ and $k^{-9/3}$. The inset shows a log-linear plot of $E(k)$ versus $k\eta$.

1) Kaneda, Y. *et al.*, Phys. Fluids, **15**, L21 (2003)
2) Tsuji, Y., Phys. Fluids, **16**, L43 (2004)