§21. Simulation Study for Technology Development to Make Reflectometer Highly Accurate

Hojo, H. (Univ. Tsukuba),
Mase, A. (Kyushu Univ.),
Kawahata, K.

For burning plasma such as ITER, electron temperature is expected to be several ten's keV. For such plasma, the relativistic effect of electrons becomes important in fusion researches. We study the effects of the relativistic correction of electron mass on microwave diagnostics [1-4]. The dispersion relation of ordinary (O) mode for a relativistic Maxwellian plasma is given by

$$\frac{kc}{\omega} = N = \left[1 - \frac{1}{A}\left(\frac{\omega_{pe}}{\omega}\right)^2\right]^{1/2}, \tag{1}$$

where A denotes the relativistic correction of electron mass for O-mode cutoff, and is given by, for $\rho << 1$,

$$A = \frac{3K_2(\rho)}{\rho^2 \int_0^\infty dp\left(p^4/\gamma^2\right)e^{-\rho\gamma}} \simeq 1 + \frac{5}{2\rho}, \tag{2}$$

with $\rho = m_e c^2/T_e$, $\gamma = (1+p^2)^{1/2}$, $p = |\boldsymbol{p}|/(m_e c)$, m_e the electron mass, c the light speed, T_e the electron temperature, and $K_2(\rho)$ is the modified Bessel function. In Ref.3, the relativistic correction $A = (1+5/\rho)^{1/2}$ is proposed, which is approximately equal to eq.(2) for $\rho >> 1$. We see that $A \to 1$ as $T_e \to 0$. We show A (solid line) as a function of T_e in Fig.1, where $1+5/2\rho$ (dashed line), and $(1+5/\rho)^{1/2}$ (dot-dashed line) are also shown. We see that $1+5/2\rho$ is in good agreement with the exact form A up to 60keV of T_e.

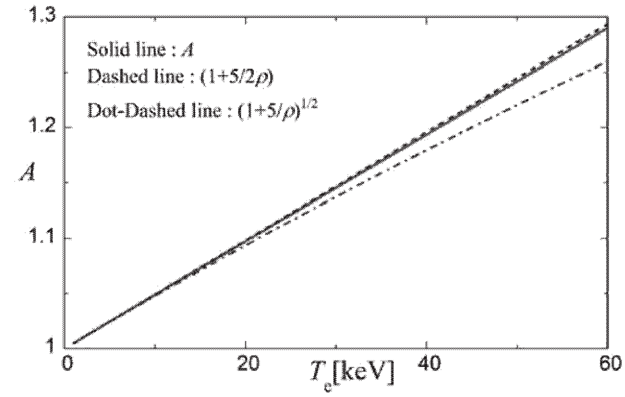

Fig.1 A (solid line), $1+5/2\rho$ (dashed line), and $(1+5/\rho)^{1/2}$ (dot-dashed line) are also shown.

In interferometry, the phase difference in plasma and vacuum propagation is important in the density profile reconstruction, and is given by

$$\phi = \int_{y_1}^{y_2}(k_0 - k)dy = \frac{2\pi}{\lambda}\int_{y_1}^{y_2}(1-N)dy, \tag{3}$$

where $k_0 = \omega/c = 2\pi/\lambda$ is wavenumber in vacuum. When $\omega >> \omega_{pe}$, eq.(3) is approximated by

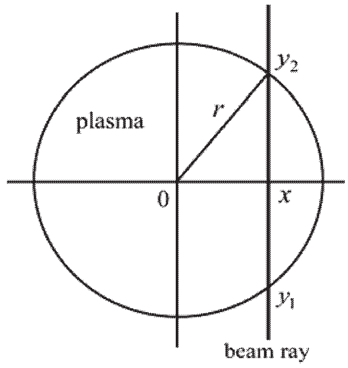

Fig. 2 Interferometer configuration

$$\phi(x) \simeq \frac{\pi}{\lambda n_c}\int_{y_1}^{y_2}\frac{n(r)}{A}dy = \frac{\pi}{\lambda n_c}\int_x^a \frac{n(r)}{A}\frac{rdr}{\sqrt{r^2 - x^2}}, \quad r > x, \tag{4}$$

where $n_c = \omega^2 m_e \varepsilon_0/e^2$ and a is a plasma–vacuum boundary (see Figure 2). If we assume a parabolic (axi-symmetric) density profile given by

$$n(r) = n_0\left[1 - \left(\frac{r}{a}\right)^2\right], \tag{5}$$

The phase difference $\phi(x)$ is reduced to

$$\phi(x) = \frac{4\pi a}{3\lambda}\frac{n_0}{An_c}\left[1 - \left(\frac{x}{a}\right)^2\right]^{3/2}. \tag{6}$$

The phase difference becomes small due to the relativistic mass correction of electron. From the assumption of axi-symmetric density profile, we can reconstruct the density profile with use of the Abel inversion equation, which is given by

$$n(r) = -\frac{\lambda n_c}{\pi^2}\int_r^a \frac{d\phi}{dx}\frac{dx}{\sqrt{x^2 - r^2}}, \quad x > r. \tag{7}$$

In this case, if we substitute eq.(6) into eq.(7), we obtain

$$n(r) = \frac{n_0}{A}\left[1 - \left(\frac{r}{a}\right)^2\right]. \tag{8}$$

The reconstructed density profile of eq.(8) is small due to the relativistic mass correction of electron by the factor A, as compared with eq.(5). It is clear the reason for this discrepancy in the density profile. That is, we did not take into account the relativistic mass correction of electron in the Abel inversion equation of eq.(7).

The Abel inversion equation with the relativistic mass correction of electron is given by

$$n(r) = -A\frac{\lambda n_c}{\pi^2}\int_r^a \frac{d\phi}{dx}\frac{dx}{\sqrt{x^2 - r^2}}, \quad x > r, \tag{7-1}$$

in this case, we can obtain the correct density profile of eq.(5) in place of eq.(8) as the reconstructed density profile. The relativistic correction is also important in reflectometry.

[1] D. B. Batchelor et al., Phys. Fluids **27** (1984) 2835.
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