§18. Anomalous Diffusion and Multiple-periodic Accelerator Modes in the Standard Map

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In order to achieve confinement of high temperature plasma in toroidal magnetic devices, it is crucial to have good knowledge of magnetic surface. It is the central issues of controlled fusion research to understand the long time behavior of plasma particles confined in the magnetic vessels1). Since the configuration of magnetic field lines is described in terms of the Hamiltonian canonical equations, many studies of the magnetic surface have been carried out by adopting the methods of nonlinear dynamic theory 2) and 3). In particular, the Poincare's mapping approach is useful to deal with the conserved Hamiltonian system of a few degrees of freedom.

The magnetic surface of toroidal configuration such as those of tokamaks, sterallators and other helical devices has been analyzed by full use of the two dimensional area preserving maps. Almost all of two dimensional area preserving maps are reduced to the standard map by local linear approximation,

$$\begin{aligned} q_{n+1} &= q_n + p_{n+1} \\ p_{n+1} &= p_n + F(q_n) \equiv p_n + A\sin(2\pi q_n) \end{aligned} \qquad (1)$$

When the nonlinear parameter A is sufficiently small, orbits described by (1) encircles the fixed point ($q_0 = 0, p_0 = 0$) with closed curves representing regular motion. Greene 4) has determined the critical value of A at which the last invariant surface called KAM surface is destroyed, and the system undergoes transition to the state of global chaos.

The stochastic properties of the system in the global chaos is best characterized by the stochastic diffusion of the orbits in the phase space. Numerical observation of the mean square fluctuation of the momentum variables can define the diffusion coefficient. As for the standard map (1), the characteristic function gives rise to asymptotic expression of the diffusion coefficient 5),

$$D = \frac{A^2}{4}\frac{1 - 2J_1^2(2\pi A) - J_2^2(2\pi A) + 2J_3^2(2\pi A)}{(1 + J_2(2\pi A))^2} \qquad (2)$$

where $J(x)$ is the n-th order Bessel function. Although 2) has been derived under the asymptotic condition of $2\pi A \gg 1$, the numerically observed diffusion processes are consistent with this theoretical prediction, except in the region of

$$|l| < A < \left(1 + \left(\frac{2}{\pi l}\right)^2\right)^{\frac{1}{2}} |l| \qquad (3)$$

where the fundamental accelerator modes

$$q_0 = \frac{1}{2\pi}\sin^{-1}\left(\frac{l}{A}\right) \qquad p_0 = 0 \qquad (4)$$

are stable. Here, l is an integer measuring the size of acceleration in momentum direction. Surrounding this fundamental accelerator mode, there occur the Poincare-Birkhoff bifurcation to generate higher order periodic accelerator modes.

In numerical observation of the stochastic diffusion 6), we have found that even in small values of the nonlinear parameter A, there appear many sharp peaks of anomalous enhancement of diffusion, where the fundamental accelerator mode can not exist. These peaks of anomalous diffusion are attributed to multiple-periodic accelerator mode which can exist even if the nonlinear parameter A is smaller than 1. The purpose of the present paper is to develop quantitative analysis of the structure, and the stability of the period-3 and the period-5 accelerator modes, and to give quantitative account of the numerically observed diffusion coefficient.

In the present studies of the multi-periodic accelerator mode of the standard map, we have determined quantitatively the regions of the anomalous enhancemen of diffusion with the contribution of the accelerator modes. It is very interesting to notice that the accelerator modes of large periodicity manifest their contribution at smaller nonlinear parameter A. In the present analysis, we have some indication of observing the contribution of the period-7 accelerator mode at the value of $A = 0.5$. It is reserved for future studies to account quantitatively the stickiness around the regular orbits, and to explore the scaling law of the transport property in the system described by the standard map.

References

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