§25. Study on Universality and Singularity of NS and MHD Turbulence at Asymptotic State by Using Massive Parallel Computation

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Huge transport power and strong fluctuations are characteristics of turbulence. Anomalous fluctuations of velocity field drives more the fluctuations and singular nature of the scalar and magnetic field. An important nondimensional parameter in the problem of the scalar transfer in fluid and magneto hydrodynamic turbulence is the Schmidt number $Sc = \nu/\kappa$ (or the magnetic Prandtl number) the ratio of the molecular viscosity to the molecular diffusivity. When $Sc = O(1)$, the dissipation lengths for the velocity and scalar are of the same order, but for larger Schmidt number, the diffusive length $\overline{\eta}_B$ become smaller than the Kolmogorov length $\overline{\eta}$ according to $\overline{\eta}_B = Sc^{-1/2}\overline{\eta}$. Batchelor obtained the spectrum of scalar variance in the viscous-convective and far diffusive ranges beyond $1/\overline{\eta}$ as

$$E_\theta^B(k) = C_B\overline{\chi}(\overline{\epsilon}/\nu)^{-1/2}k^{-1}\exp\left(-\kappa k^2/|\gamma_*|\right)$$

which rolls off rapidly in the far diffusive range, where γ_* is a representative value of the smallest eigenvalue of the strain tensor [1]. Kraichnan studied the scalar spectrum for the rapidly changing random velocity field and obtained

$$E_\theta^K(k) = C_B\chi(\overline{\epsilon}/\nu)^{-1/2}k^{-1}(1+\sqrt{6C_B}k\eta_B) \times \exp\left(-\sqrt{6C_B}k\eta_B\right),$$

where $\overline{\epsilon}$ and $\overline{\chi}$ are the mean energy dissipation rate, the mean scalar variance dissipation rate, respectively and C_B is a nondimensional constant [2]. The difference between two theories lies in the facts that Bathelor's theory is a mean field theory in the sense that γ_* is assumed to be constant and estimated as $|\gamma_*| = C_B(\overline{\epsilon}/\nu)^{1/2}$ where $(\overline{\epsilon}/\nu)^{1/2}$ is the mean strain rate, while the effects of fluctuations of the strain are included in Kraichnan's theory.

We have numerically examined the scalar spectrum in the viscous-convective and far diffusive ranges for very high Schmidt numbers of $Sc = 200, 1000$ by using the hybrid code which uses the spectral method for the incompressible velocity and the combined compact finite difference method for the passive scalar. For the case of high Schmidt number the high Reynolds number is not necessary condition. We set R_λ about 42 so that the number of grid points for the velocity was 256^3 while 2048^3 for the scalar to resolve the small scales of the passive scalar in the far diffusive range. In order to obtain well converged statistics we integrated the equations longer than 72 large eddy turn over times for the time average. This remarkably long time computation was achieved due to the high performance of the hybrid computation. The Batchelor constant is found to be $C_B = 5.7$ which is larger than 4.9 by Donzis et al. [3] . The spectrum in the far diffusive range is exponential as seen in Fig.1 which is consistent with the Kraichnan spectrum. We also computed the probability density functions (PDFs) for the eigenvalues $\lambda_3 < \lambda_2 < \lambda_1$ of the strain field as in Fig.2. The PDF of λ_3 is strongly non-Gaussian. Although the Batchelor spectrum for $E_\theta(k)$ decays in Gaussian form, it is reasonable to replace the mean strain rate γ_* by the fluctuating negative strain rate λ_3 and to take an average over the PDF of λ_3 as

$$E_\theta^{\mathrm{av}}(k) = \int_{-\infty}^{0} P(\lambda_3)E_\theta^B(k,\lambda_3)d\lambda_3.$$

The averaged spectrum $E_\theta^{\mathrm{av}}(k)$ thus obtained is compared to DNS spectrum in Fig.1, which shows that the exponential decay of the averaged spectrum well agrees with DNS data. This means that the far tail of the scalar spectrum is strongly governed by the strong fluctuation of the strain rate, the intermittency.

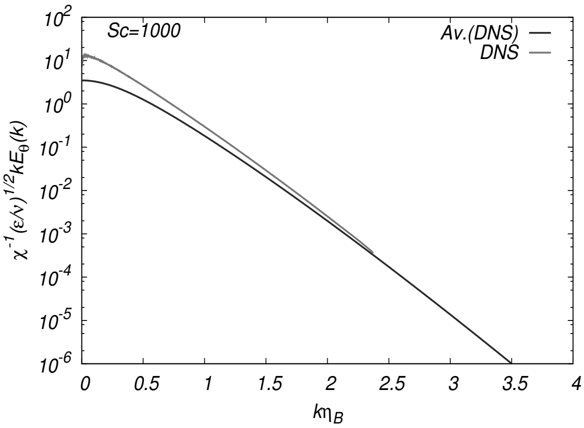

Fig. 1: Comparison of $E_\theta^{\mathrm{av}}(k)$ (long line) averaged over PDF of the most compressible eigenvalue of the strain tensor and $E_\theta^{\mathrm{DNS}}(k)$ (short line) computed by DNS at $R_\lambda = 42$.

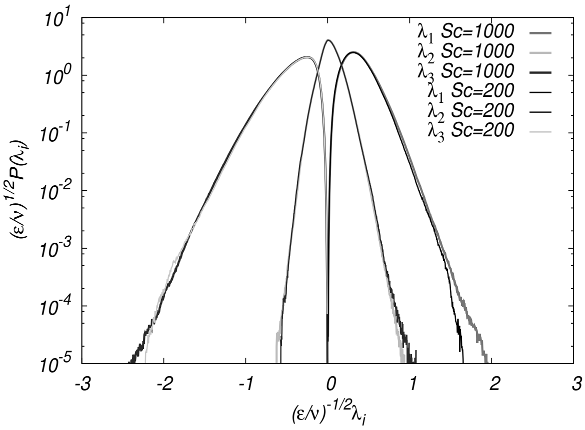

Fig. 2: PDF of eigenvalues of the strain tensor.at $R_\lambda = 42$.

1) Batchelor, G. K. J. Fluid Mech., (1959) **5**, 113-133.

2) Kraichnan, R. H. J. Fluid Mech., (1974) **64**, pp.737-762.

3) Donzis, D. A., Sreenivasan, K. R., and Yeung, P. K. Flow, Turb. and Combust., (2010) **85**, pp.549-566.