§55. Response and Stability of MHD and NS Turbulences

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When compared to the study of the statistical properties of NS and MHD turbulences such as energy spectrum, scaling exponents, stability of turbulence has attracted little attention. Turbulence is a random phenomena, highly mixing, and very sensitive to small disturbances. When the turbulence is disturbed, the phase orbit of the NS and/or MHD solution in the infinite dimensional phase space would be much different from the undisturbed NS or MHD would have. However, it is unrealistic to trace the orbit in the phase space. Rather, it is more appropriate and physically important to consider the response of the statistical quantities of the turbulence.

It is well known that study of response of a system in thermal equilibrium was essential to construct the nonequilibrium statistical mechanics, and to understanding of the phenomena. For example, the response of a canonical ensemble to change of temperature is described as the specific heat which is related to the fluctuations of the energy. In the nonequilibrium statistical mechanics, generally, it is well known that response of a system is related to the correlation of the thermal fluctuations, the fluctuation-dissipation theorem. By analogy, we hope that to examine the response of MHD and NS turbulences in the statistical sense leads to deeper understanding of turbulence dynamics and would be helpful for construction of statistical mechanics of turbulences.

When the NS turbulence in a steady state is disturbed, it would restore the equilibrium state if the turbulence is statistically stable. We have examined the way of return to the stationary state by using high resolution DNS. The velocity amplitudes in the wavenumber spacet is disturbed while the phases are unchanged, so that the energy spectrum is distorted slightly. Actually the slope of the energy spectrum is slightly increased (decreased) but with the same total energy, that is $E_{purtb}(k) = (k/k_*)^\delta E(k)$, where $E(k)$ is the stationary spectrum before the disturbance is added and $|\delta| \ll 1$. k_* is so chosen that $\int E_{purtb}(k)dk = \int E(k)dk$.

The deviation of the disturbed spectrum from the equilibrium state is defined as

$$W_E(k,t-s) = E_{purtb}(k,t-s) - E(k).$$

We have computed the relaxation of $W_E(k,t-s)$ by DNS. Figure 1 shows the evolution of $W_E(k,t-s)$ for $\delta = 0.1$ at $R_\lambda = 235$. At wavenumbers higher than $k_* = 4$, $W_E(k,t-s)$ once grows and then decays with oscillation. This is due to enhanced energy transfer by increase of total shear by all the Fourier components below k. Figure 2 shows each band relaxation as functions of time difference which is defined as

$$C(k,t-s) = W_E(k,t-s)/W_E(k,0)$$

Time is scaled by the total strain $\mu(k) = \sqrt{\int_0^k p^2 E(p)dp}$. Behavior of the energy transfer through k is found to be consistent with the above explanation (figure not shown).

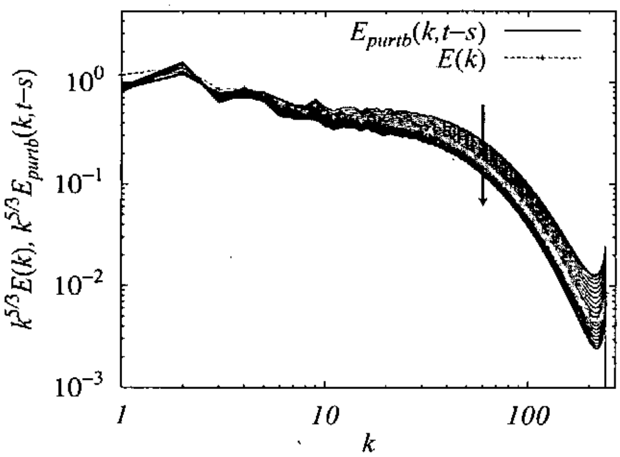

Fig.1 Relaxation of the purturbed energy spectra toward the stationary state. $R_\lambda = 235, \Delta = 0.1, k_* = 4$.

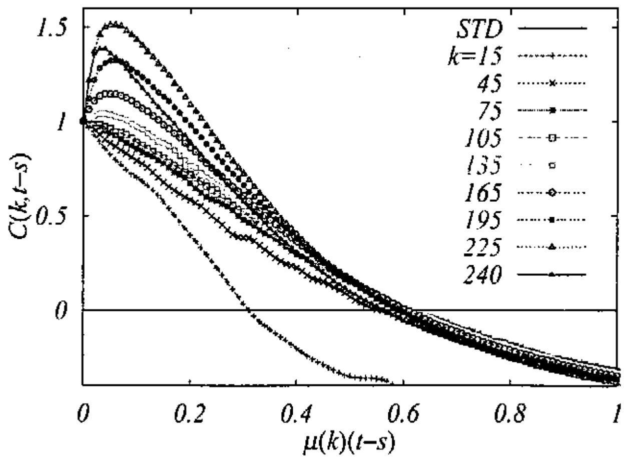

Fig.2 Relaxation of each band correlation function $C(k,t-s) = W_E(k,t-s)/W_E(k,0)$.