§6. Study of Energy Transfer in NS and MHD Turbulence by Using Massive Parallel Computation

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Variation of fluctuations with decrease of scales is one of the important aspects of anomalous fluctuations observed in NS and MHD turbulence. Keys to the understanding of the fluctuations are singularities existing in the turbulent fields which are generated by stepwise or direct transfer of the energy from large to small scales through the inertial ranges. Also the conserved quantities in the inviscid truncated system which has a finite wavenumber truncation and zero viscosity and zero magnetic diffusivity. The singularity structure and conserved quantities strongly depend on the spatial dimensions. Although the number and kinds of the conserved quantities differ in even and odd spatial dimensions, the second order quantities in field amplitudes play key roles in direct numerical simulation (DNS) in turbulence. For example, the energy and enstrophy in 2D and the energy in 3D and 4D. We have done DNSs of the decaying NS turbulence in 3D and 4D, examined the energy decay rate, probability density function of the eigen values of the strain tensor, Kármán-Howarth-Kolmogorov (KHK) equation. Also the analysis has been extended to 5D case and the decay rate of the total energy and the transfer rate of the energy to high wavenumber components were analyzed.

The energy spectrum is normalized in such as way that the initial velocity field in d-dimensions is solenoidal and Gaussian random and each component of the kinetic energy is the same;

$$E = \frac{d}{2}u'^2 = \int_0^\infty E(k)dk, \quad R_\lambda = \frac{u'\lambda}{\nu}.$$

KHK equation is given by

$$\frac{D_{LLL}(r)}{\bar{\epsilon}^{(d)}r} = -\frac{12}{d(d+2)} + \frac{6\nu}{\bar{\epsilon}^{(d)}r}\partial D_{LL}(r)/\partial r - \frac{3}{\bar{\epsilon}^{(d)}r r^{d+1}}\int_0^r \partial D_{LL}(r')/\partial t (r')^{d+1}dr'$$

where $D_{LL}(r)$ and $D_{LLL}(r)$ are the second and third order structure functions of the longitudinal velocity increments, and $\bar{\epsilon}^{(d)}$ denotes the energy dissipation rate of the energy in d-dimensions. Figures 1 and 2 show the time evolution of the total energy and total enstrophy which are normalized by their initial values. As was inferred from the study in 3D and 4D, the energy transfer in 5D became more efficient than in 3D and 4D. It was also observed in Fig.3 that KHK equation was also satisfied. These findings suggest that the fundamental mechanisms of the energy transfer of the NS turbulence in 5D is very much similar to that in 3D but the energy transfer is more efficient.

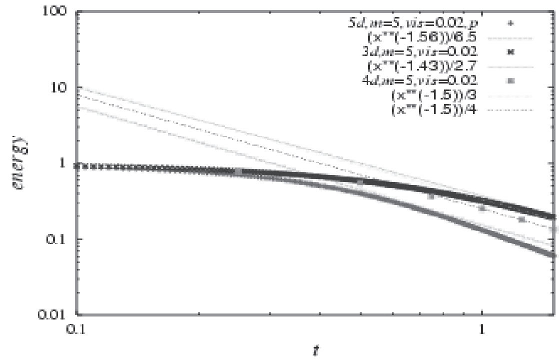

FIG. 1: Comparison of decay of the energy $E^{(d)}(t)/E^{(d)}(0)$ for $d = 3, 4, 5$. The decay becomes faster with d.

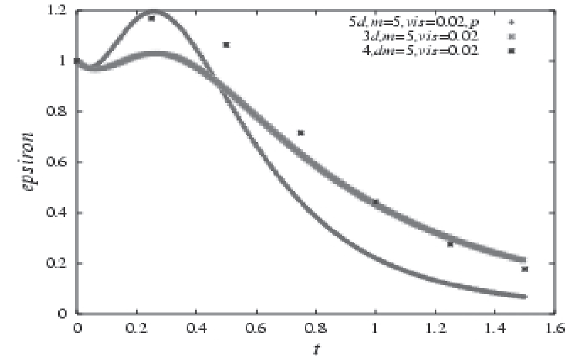

FIG. 2: Evolution of the energy dissipation $\bar{\epsilon}^{(d)}(t)/\bar{\epsilon}^{(d)}(0)$ for $d = 3, 4, 5$.

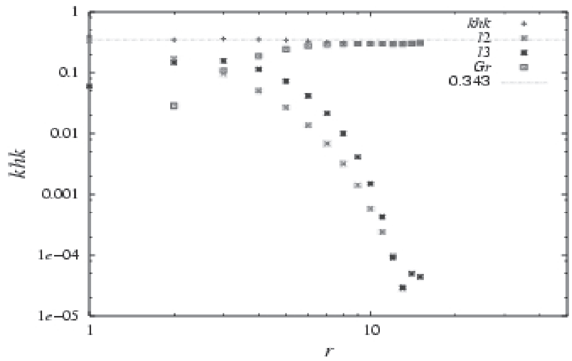

FIG. 3: Kármán-Howarth-Kolmogorov equation. Each term is divided by $\bar{\epsilon}^{(d)}r$. The horizontal line shows $12/35 \approx 0.343$.

[1] E. Suzuki, T. Nakano, N. Takahashi, and T. Gotoh, Phys. Fluids **17**, 081702 (2005).

[2] T. Gotoh, Y. Watanabe, Y. Shiga, T. Nakano, and E. Suzuki, Phys. Rev. E**75**, 016310 (2007).