§30. Analysis of the Energy Transfer in NS and MHD Turbulence by Using Massive DNS

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It has long been believed that the energy spectrum in the inertial range of MHD turbulence obeys Iroshinikov-Kraichnan scaling $E(k) \propto k^{-3/2}$ [1]. Some of recent numerical simulations, however, have found that the Kolmogorov scaling of the energy spectrum holds also for the MHD turbulence. To the author's view, still it is far from the definite conclusion because the width of wavenumber range for the Kolmogorv scaling is not long enough. In order to get hints about this problem, on one hand avoiding the resolution limit of the numerical simulation, we have numerically studied the local or nonlocal nature of the energy transfer among the kinetic or magnetic components in the wavenumeber space. Let us write the equations for the energy spectra as

$$\partial E^K(k,t)/\partial t = T^K(k,t) - D^K(k,t) = \sum_q (T_{uu}(k,q,t) + T_{ub}(k,q,t)) - D^K(k,t),$$

$$\partial E^M(k,t)/\partial t = T^M(k,t) - D^M(k,t) = \sum_q (T_{bu}(k,q,t) + T_{bb}(k,q,t)) - D^M(k,t),$$

where $D^\alpha(k,t), (\alpha = K, M)$ denotes the dissipation spectrum and $T^\alpha(k,t)$ for the energy transfer due to the nonlinear interactions, in which the sum over the third wavenumber $\boldsymbol{p}$ in the triad interaction $(T^\alpha(\boldsymbol{k},\boldsymbol{p},\boldsymbol{q}), \boldsymbol{k} = \boldsymbol{p} + \boldsymbol{q})$ is taken for easiness of analysis. For example $T_{ub}(k,q)$ stands for contributions of $\boldsymbol{b}(\boldsymbol{q})$ to $\boldsymbol{u}(\boldsymbol{k})$.

Direct numerical simulations were done for Elsässer variables $\boldsymbol{z}^\pm = \boldsymbol{u} \pm \boldsymbol{b}$ with unit magnetic Prandtl number. Gaussian random fields with or without kinetic and/or magnetic helicity were initially generated and then evolved freely without any forcing. It was found that the magnetic helicity was important in the evolution of the energy spectrum. When H^M is finite, the energy cascades towards low wavenumbers. The energy decay is found to be $E(t) \propto t^{-\beta}, \beta = 1.2 (H^M = 0), 0.68 (H^M \neq 0)$ and becames slower when $H^M \neq 0$. Hatori has shown that $\beta = 2/3$ for $H^M \neq 0$ [2]. Figures 1 and 2 show comparison of $T_{uu}(k,q)$ and $T_{ub}(k,q)$ at $q = 40$. $T_{uu}(k,q)$ is local in wavenumbers, in the sense that the transfer occurs at wavenumbers k nearby q. On the other hand $T_{ub}(k,q)$ is nonlocal in that for almost all wavenumebrs of $k < q$, T_{ub} remains uniformly constant and negative by reflecting the Lorentz force. Also it was found that the transfer in T_{bu} is nonlocal while that of T_{bb} is local. These facts suggest that the scaling exponent of the energy spectrum depends on relative strength between E^K and E^M and on the relative strength between (T_{uu}, T_{bb}) and (T_{ub}, T_{bu}).

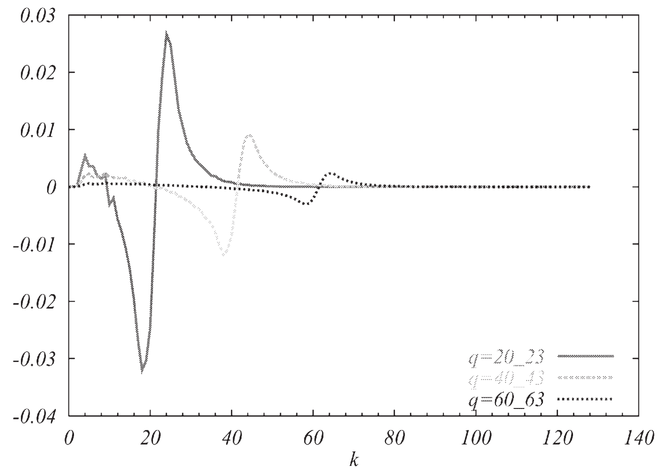

Fig.1 Plot of $T_{uu}(k,q)$ for $q = 20-23, 40-43, 60-63$ when $H^M = 0$ at $t = 0$. The energy is transferred locally.

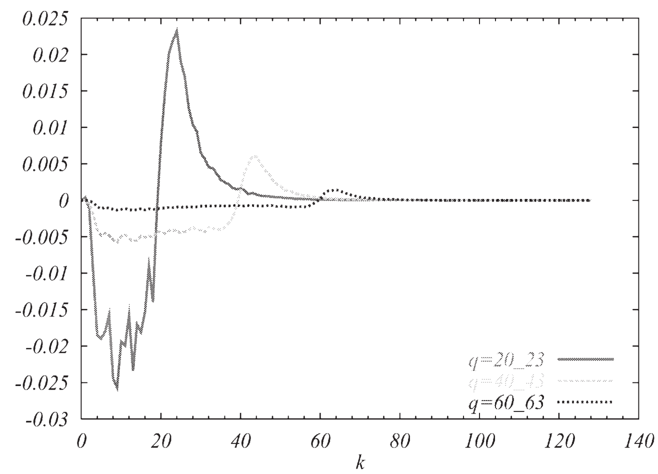

Fig.2 Plot of $T_{ub}(k,q)$ for $q = 20-23, 40-43, 60-63$ when $H^M = 0$ at $t = 0$. The energy is transferred nonlocally.

References

1) D. Biskamp, *"Magnetohydrodynamic turbulence"*, Cambridge University Press, (2003).

2) T. Hatori, J. Phys. Soc. Jpn. **53**, 2539 (1984).

3) H. Kurosawa. *Energy transfer among wavenumbers in magnetohydrodynamic turbulence*, NIT Master Thesis (in Japanese) (2006).