§30. Probability Density Functions and Structure of Singularities in Non-Dissipative Fields in MHD and NS Turbulences

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Recent studies on a passive scalar convected by an artificial random solenoidal velocity field (Kraichnan model) have found that the scaling of the high order structure functions of the scalar increments tends to the normal scaling in the large spatial dimensions [1]. Also in the equilibrium statistical mechanics, fluctuations near phase transition point obey the Gaussian in large dimensions too, and the mean field theory works well. Then it is quite natural to ask whether the intermittency of turbulence changes when the spatial dimensions becomes larger than 3. In 2-dimensions there is no vortex stretching so that the fair comparison with 3-dimensional turbulence is difficult. On the other hand, in 4-dimensions there exists the streching of the velocity 2-form and thus the energy cascade towards large wavenumbers can be expected [2].

NS equation in 4-dimensions is written in terms of the velocity 2-form $\Omega_{ij} = \partial u_j/\partial x_i - \partial u_i/\partial x_j$ as

$$\partial u_i/\partial t + u_k \Omega_{ki} = -\partial_i P + \nu \nabla^2 u_i, \qquad \partial_i u_i = 0$$

Under the periodic boundary conditions, we have numerically integrated the above equation by using the spectral method and 4th order Runge-Kutta-Gill method. With the same energy per each direction, the comparison of the 4-dimensional turbulence with that in 3-dimensions was done at similar Reynolds numbers.

Figure 1 shows the evolution of the average rate of the energy dissipation $dE^{(d)}/dt = -\epsilon^{(d)}$ normalized by their initial values. It follows that the energy transfer in 4d is more efficient than in 3d, so that the decay of the total energy in 4d is faster than in 3d. The time needed for formation of the singular structure is shorter in 4d and and its strength is bigger than in 3d. Correspondingly to this, the skewness and flatness factors of the longitudinal velocity gradient in 4d are bigger than in 3d. Figures 2 and 3 show the distribution of the enstrophy $\sum_{ij} \Omega_{ij}^2$ visualized by the same threshold value. The high intensity domains of the enstrophy in 4 dimensions are less space filling when compared to those in 3 dimensions. This is a dimension effect that the number of terms becomes larger in high space dimension so that the fluctuations become milder with increase of dimensions.

References 1) G. Falkovich, K. Gawedzki and M. Vergassola, Rev. Mod. Phys. **73**, 913 (2001).
2) E. Suzuki, T. Nakano, N. Takahashi, and T. Gotoh, Phys. Fluids **17**, 081702 (2005).
3) Y. Watanabe, *Statistics of the large scale velocity field in turbulence*, NIT Master Thesis (in Japanese) (2006).

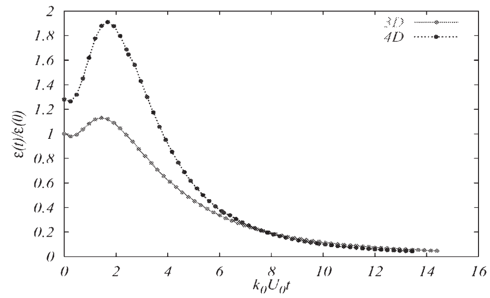

Fig.1 $\epsilon^{(d)}(t)/\epsilon^{(d)}(0)$

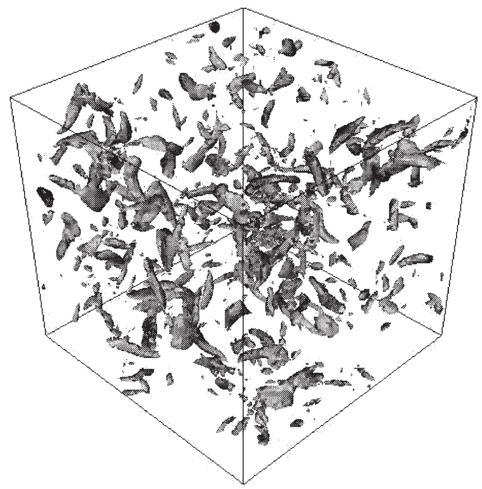

Fig.2 $\sum_{ij}[\Omega_{ij}^{(3)}(\boldsymbol{x})]^2 > 6\sigma(\sum_{ij}[\Omega_{ij}^{(3)}]^2)$ where σ is the standard deviation.

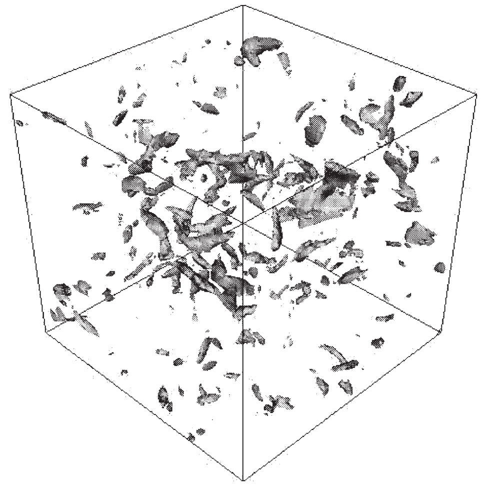

Fig.3 $\sum_{ij}[\Omega_{ij}^{(4)}(x_1, x_2, x_3, 0)]^2 > 6\sigma(\sum_{ij}[\Omega_{ij}^{(4)}]^2)$.