§37. Mechanism of Energy Cascade in NS and MHD Turbulence by Using Parallel Massive Direct Numerical Simulation

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Direct numerical simulation (DNS) of Navier-Stokes, passive scalar and MHD turbulences requires a huge amount of computational resources. In order to attain high (magnetic) Reynolds number as high as possible, it is quite common to set the minimum grid space Δ in DNS to be the mean Kolmogorov length $\overline{\eta}$. In the case of spectral method, this means $K_{max}\overline{\eta} > 1$, where K_{max} is the maximum wavenumber. However, it is well known that fluctuations in the energy cascade from large to small scales of motion grow as the cascade proceeds, and result in strong fluctuations in the energy dissipation ϵ. Then the local Kolmogorov length defined by $\eta(\boldsymbol{x},t) = (\nu^3/\epsilon(\boldsymbol{x},t))^{1/4}$ also fluctuates in space and time, which in turn implies that the usual criterion for the spatial resolution is not enough to resolve the singularity. There are no knowledge to what extent the inertial range dynamics and statistics are affected by the insufficient accuracy in the dissipation range, and it is indispensable for the future high Reynolds number turbulence DNS to know degree of the contamination or modification in this range.

From the above view point, we have analysed the effects of the resolution on the intermittency of the passive scalar convected by NS turbulence. We have done DNSs with different resolutions while keeping Taylor's micro scale Reynolds number constant. They are L1($N = 256, K_{max}\overline{\eta} = 1.0$), L2($N = 512, K_{max}\overline{\eta} = 2.0$), L3($N = 1024, K_{max}\overline{\eta} = 3.8$) at $R_\lambda = 180$, and H1($N = 1024, K_{max}\overline{\eta} = 1.06$), H2($N = 2048, K_{max}\overline{\eta} = 2.19$) at $R_\lambda = 420$. Schmidt number $Sc = \nu/\kappa$ was fixed as unity.

It was found that tails of the probability density function (PDF) become shorter, while the energy spectrum is unchanged except at wavenumbers near K_{max} as shown in Fig.1. The same is true for the spectrum for the passive scalar variances. Reynolds number dependence of approach to the 4/3 law for the scalar flux defined by $S_3^{\theta L}(r) = \langle (u(\boldsymbol{x}+r\boldsymbol{e}_1) - u(\boldsymbol{x}))(\theta(\boldsymbol{x}+r\boldsymbol{e}_1) - \theta(\boldsymbol{x}))^2 \rangle$ was examined, and it was found that the resolution effects on this law is very small. On the other hand, high order moments of the scalar flux $S_q^{\theta L}(r) = (\overline{\chi}r)^{-q/3} S_q^{\theta L}(r)$ are found to be sensitive to the variation of the resolution as seen in Fig.2. However, it is good news that the scaling exponents of the correlation functions in the inertial range for $q \leq 8$ are insensitive (figure not shown). Implication to the case of MHD turbulence is such that degree of the sensitiveness of the magnetic field in the inertial range to the dissipation range resolution would be less than the case of the passive scalar and rather similar to the case of the velocity field because of $\mathrm{div}\boldsymbol{B} = 0$.

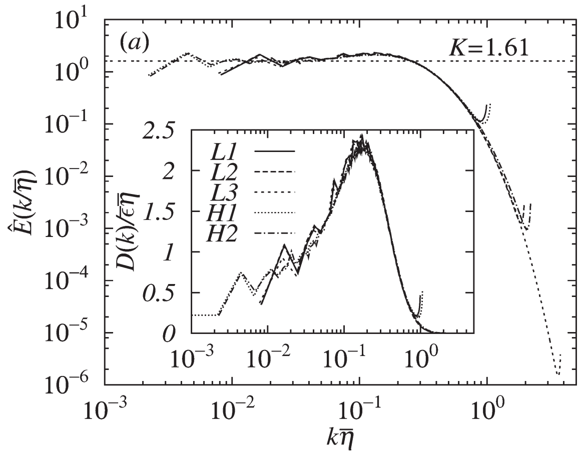

FIG. 1: Energy spectrua at $R_\lambda = 180, 420$. Inset shows the energy dissipation spectra $D(k) = 2\nu k^2 E(k)$.

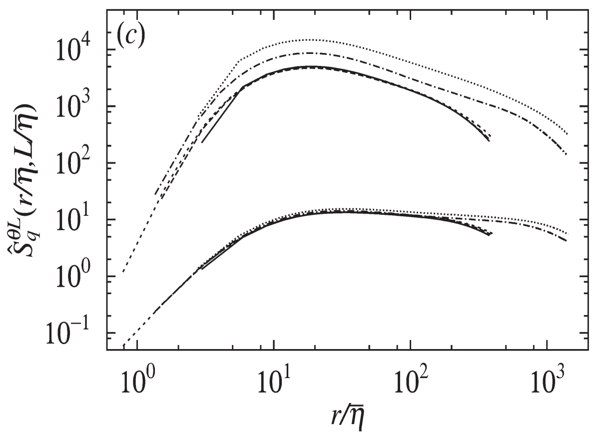

FIG. 2: Structure functions of the scalar transfer flux $S_q^{\theta L}(r)$. Curves for $q = 4$ (bottom), Curves for $q = 8$ (upper).