§16. Lagrangian Direct-interaction Approximation for Homogeneous Isotropic Turbulence

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The closure problem of homogeneous isotropic turbulence of an incompressible neutral fluid is considered by a Lagrangian direct-interaction approximation (DIA). Here, we assume the dynamics of the system is governed by the Navier-Stokes equation and the equation of continuity. Then, the basic equation is expressed symbolically as

$$\left[\partial/\partial t+\nu\right] X_i = M_{ijk}\, X_j\, X_k \;, \qquad (1)$$

where X_i denotes the Fourier component of the Eulerian velocity field and ν expresses the viscosity. One of the main aims of the present study is to obtain a closed set of equations for the correlation function $V_{in}=\overline{X_i X_n}$, which is governed by an equation such as

$$\left[\partial/\partial t+\nu\right] V_{in} = M_{ijk}\, \overline{X_j\, X_k\, X_n} \;, \qquad (2)$$

and a few functions (e.g. the response function $G_{in}=\delta X_i/\delta X_n$). Note that we actually employ the Lagrangian velocity correlation function instead of the Eulerian velocity correlation to guarantee the invariance under the Galilean transformation. In order to construct closed equations, we must express the three-component correlation, which exists on the right-hand side of (2), in terms of V_{in} and G_{in}.

For this purpose, we have developed DIA, which was originally proposed by Kraichnan [1]. This approximation is based upon the following two ideas. (I) The correlation between components X_{i_0}, X_{j_0} and X_{k_0} mainly originates from the direct interaction term on the right-hand side of (1). For example, the term of $M_{i_0 j_0 k_0}X_{j_0}X_{k_0}$ predominantly yields the dependence of X_{j_0} and X_{k_0} on X_{i_0}. Hence, if we extract the direct interaction term between the three components, the correlation between them may weaken. (II) An influence of such an extraction of only a single direct interaction should be negligible, if the number of the degrees of freedom of the system is large enough.

Some straightforward calculations based upon the above two ideas lead a closed set of equations for the correlation and the response functions (see ref.[2] for the detail). It is shown from the closed equations that the universal form of the energy spectrum function, which is simply related with the correlation function, is common in the stationary and the decaying turbulence. Then, we have solved numerically the equation by an iteration method, and the result for the one-dimensional longitudinal energy spectrum E is shown in Fig.1 together with various kinds of experimental data. The agreement in the universal range of the wavenumber k is excellent; the deviation in the lower wavenumber range (non-universal range) is due to the large-scale structures which depend on the kind of turbulence. Such an agreement is quite surprising, because the Lagrangian DIA is a theory without any adjustable parameters.

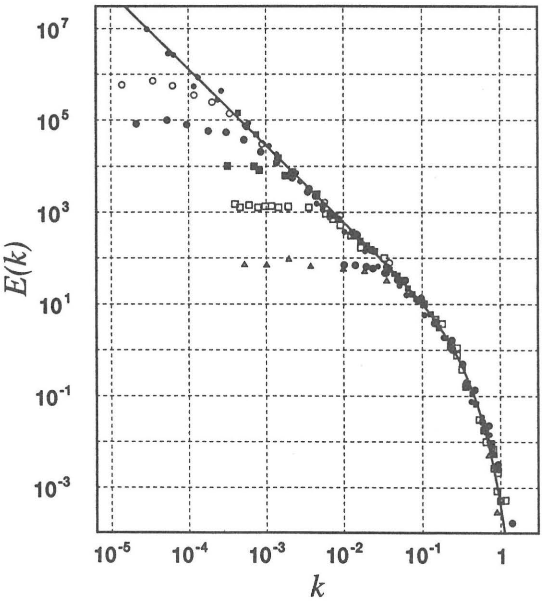

Fig. 1. One-dimensional longitudinal energy spectrum. Symbols and solid line respectively denote experimental data and the theoretical evaluation by the Lagrangian DIA.

References

1) Kraichnan, R. H., J. Fluid Mech. 5 (1959) 497.

2) Kida, S. and Goto, S., J. Fluid Mech. (in print).