§26. Lagrangian Direct-interaction Approximation for a Passive Scalar Field

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It is interesting to investigate statistical property of a scalar field $\theta(\boldsymbol{x}, t)$ such as temperature, particle concentration, etc., which is diffused and passively advected by turbulent flow as

$$\frac{\partial \theta}{\partial t} + u_i \frac{\partial \theta}{\partial x_i} = \kappa \frac{\partial^2 \theta}{\partial x_i \partial x_i}. \quad (1)$$

Here, κ is the diffusion coefficient and $u_i(\boldsymbol{x}, t)$ ($\partial u_i / \partial x_i = 0$) is an incompressible turbulent velocity field governed by the Navier-Stokes equation,

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j}, \quad (2)$$

where ν is the kinematic viscosity, and p and ρ are the pressure and the constant density of the fluid, respectively. Micro-scale statistics of a passive scalar field has been studied by many researchers from phenomenological viewpoints (see §VI-10 of Ref. 1 for a review). They have pointed out that statistics of the passive scalar field strongly depends upon the Schmidt number $s = \nu/\kappa$. For example, it has been proposed that the passive scalar spectrum function $\Theta(k)$, which is defined by the Fourier transform of the two-point correlation function of $\theta(\boldsymbol{x})$, obeys different scaling laws depending upon the Schmidt number s: (i) $k^{-5/3}$ law in the inertial-advective range (irrespective of s), (ii) k^{-1} law in the viscous-advective range (for $s \gg 1$, see Fig.1) and (iii) $k^{-17/3}$ law in the inertial-diffusive range (for $s \ll 1$). Although these scaling laws are supported by experiments or numerical simulations, the phenomenologies can predict neither the universal coefficients of the scaling functions nor the functional forms of the spectrum in the whole universal wavenumber range.

A purpose of this study is to predict functional forms of the passive scalar spectrum from the basic equations (1)(2) by overcoming the closure problem of moment hierarchy. We have developed a closure theory so-called the Lagrangian direct-interaction approximation (DIA) for velocity correlation functions [2] based upon *the weak dependence principle* proposed by Kraichnan [3]. Note that the Lagrangian DIA formulation is different from those of the other DIAs. The Lagrangian DIA is formulated for the Lagrangian correlation and Lagrangian response functions in contrast with the original Eulerian DIA [3], and its formulation is much simpler than that of Kraichnan's Lagrangian DIA [4]. Since the Lagrangian DIA for the velocity correlation function was shown to be quite successful [2], we apply it to a passive scalar field to derive from (1)(2) a closed set of equations for the passive scalar spectrum function and the velocity correlation function. Then, we can analytically show that the Schmidt number dependence of solutions to the resultant closure equations is completely consistent with the phenomenologies; the three scaling functions ($k^{-5/3}$, k^{-1}, $k^{-17/3}$) are solutions to them. (See Ref.5 for a derivation of the Lagrangian DIA equations and analyses of the solutions.) Furthermore, we can evaluate functional forms of the spectrum function for arbitrary Schmidt numbers in the entire universal wavenumber range by solving the closure equations numerically. Fig.1 is an example of the results, which shows the universal form of the passive scalar spectrum in the large Schmidt number limit. We can confirm that the $k^{-5/3}$ and the k^{-1} scaling functions are solutions to the Lagrangian DIA equations.

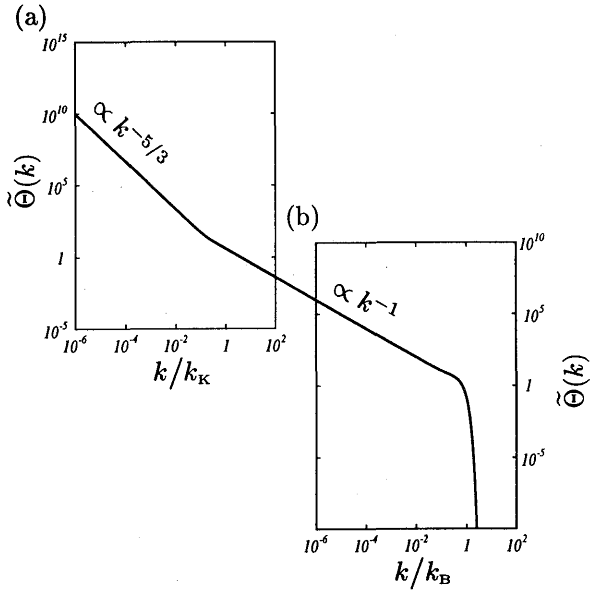

Fig.1 Non-dimensional passive scalar spectrum function $\widetilde{\Theta}(k)$ around (a) $k_K = (\epsilon/\nu^3)^{1/4}$ and (b) $k_B = k_K s^{1/2}$ (ϵ is the energy dissipation rate) in the large Schmidt number limit. The line denotes a solution to the Lagrangian DIA equations. We call the wavenumber ranges $k \ll k_K$ and $k_K \ll k \ll k_B$ the inertial-advective and viscous-advective ranges, respectively.

References

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2) Kida, S. and Goto, S., J. Fluid Mech. **345** (1997) 307.
3) Kraichnan, R. H., J. Fluid Mech. **5** (1959) 497.
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5) Goto, S. and Kida, S., Phys. Fluids (in press).