§ 25. Material Object Stretching in Turbulence

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A line (surface) which always consists of a same set of fluid particles is called a material line (surface). It has been intuitively predicted[1] and numerically confirmed[2] that the total length (area) of a material line (surface) in turbulence grows exponentially in time. Since this strong stretching is closely related with the strong turbulent mixing, which is an important problem not only theoretically but also in applications, the stretching of material objects has been intensively studied by many authors. However, the physical mechanism of neither strong mixing nor strong stretching of material objects in turbulence has been understood well.

It is well-known, on the other hand, that there exit small-scale coherent vortical structures universally in turbulence. The coherent vortical structures have tubular shapes with radius of $O(5\eta)$ and lengths ranging between η and $\mathcal{L}$. Here, η is the Kolmogorov length, the minimum length scale in isotropic turbulence, and $\mathcal{L}$ is the integral length, the maximum length scale. A typical cluster of the coherent vortical structures is shown in Fig.1. It may be observed clearly that the tubular vortices tend to align to each other in an anti-parallel manner. This strong tendency of anti-parallel alignment of coherent vortices is confirmed also quantitatively (see Ref.3).

An important inherent feature of an anti-parallel vortex pair is that there are two hyperbolic stagnation points on the cross-section which moves with the pair. Around the stagnation points, magnitudes of eigenvalues of strain tensor are large, and the streamlines on the cross-section are nearly parallel to the first eigenvector of the tensor. It is these features of an anti-parallel vortex pair that serve a physical explanation of strong stretching of material objects in turbulence, which is summarize as follows. [1] Material lines — since a material surface may be regarded as a set of material lines, the physical mechanism of their strong stretching is the same as that of lines — are likely to align to quasi-stationary streamlines, i.e., streamlines on the frame moving with a vortex pair. [2] Since the streamlines are nearly parallel to the strain-tensor eigenvectors around the hyperbolic stagnation points, and since the eigenvalues are large there, material lines are strongly stretched around the stagnation points. We plot a typical turbulent velocity field and the appearance of material lines stretching on the cross-section of a vortex pair in Fig.2, which strongly supports the above argument.

In conclusion, universally existing coherent tubular vortical structures in turbulence tend to align to each other in an anti-parallel manner, and such an anti-parallel pair of vortices sustain the strong stretching of material objects around the hyperbolic stagnation points between them.

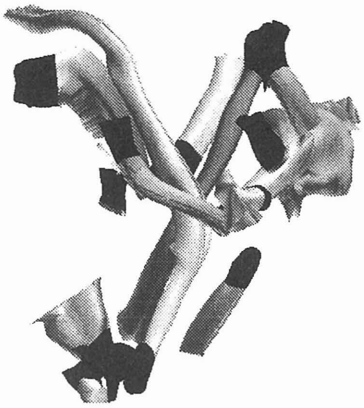

Fig.1 Cluster of small-scale coherent vortical structures in turbulence. The vorticity vector inside each vortex tube points approximately from marked end to unmarked one.

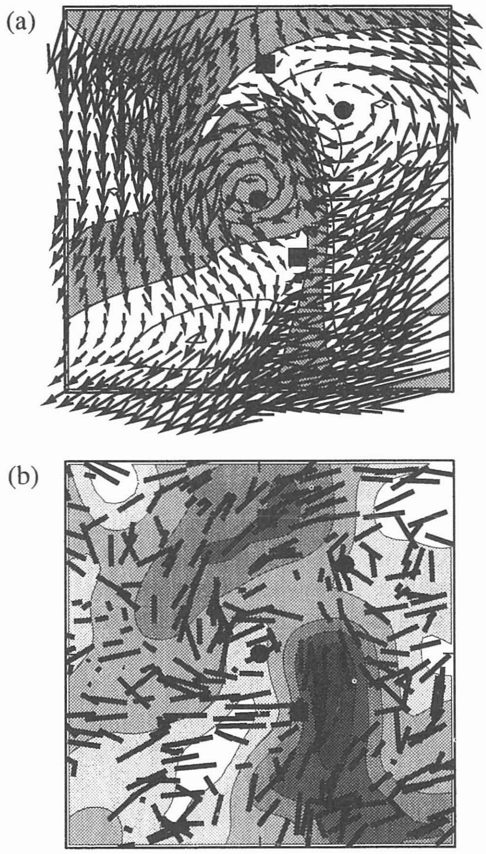

Fig.2 Cross-section of an anti-parallel vortex pair indicated by solid circles. (a) Arrows: relative velocity to the center. Hyperbolic stagnation points are denoted by solid squares. Contours: normal components of vorticity. Positive regions are shaded. (b) Lines: projections of material line elements near the cross-section. Contours: magnitude of their stretching rates.

Reference

1) Bathelor, G. K., Proc. Roy. Soc. London A **213** (1952) 349.

2) Kida, S. and Goto, 2. Phys. Fluids **14** (2002) 352.

3) Goto, S. and Kida, S. Fluid Dyn. Res. (2003) in press.