§22. Material Surface Deformation in Turbulence

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A material surface is the fictitious two-dimensional object which consists of a same set of fluid particles. In other words, a point, whose position vector $\boldsymbol{x}_p(t)$ on the surface, is advected by fluid motion according to the advection equation, $\mathrm{d}\boldsymbol{x}_p/\mathrm{d}t = \boldsymbol{u}(\boldsymbol{x}_p(t), t)$. Here, $\boldsymbol{u}(\boldsymbol{x}, t)$ is the velocity field, and is, hereafter, assumed to be governed by the Navier-Stokes equation of an incompressible fluid. Since a material surface may be regarded as a boundary of two parts of a fluid, statistics of its deformation have been intensively investigated by many authors as a foundation of the mixing by fluid motions.

If the velocity field is turbulence, a material surface is deformed in a quite complicated manner. We show a typical material surface deformed by turbulence in Fig.1, which is a result of direct numerical simulation of solving temporal evolutions of both of the velocity field and the material surface simultaneously. An important feature of this system is the exponential growth of total area, $A(t)$, of the surface. We plot temporal evolutions of the area of material surface in Fig.2 for two different Reynolds numbers, R_λ, based on the Taylor micro scale. It is seen that the areas grow exponentially as

$$A(t) = A(0) \exp\left[\, 0.3\, t/\tau_\eta \,\right], \tag{1}$$

irrespective of the Reynolds number. Here, τ_η denotes the Kolmogorov time, which is the minimum time scale of the Lagrangian motions in turbulence. The exponent

$$\gamma(t) = \frac{\mathrm{d}}{\mathrm{d}t} \log A(t) \tag{2}$$

is called the stretching rate of material surface, and has been a main research target of this system. Before our simulation, it was $0.16{\tau_\eta}^{-1}$ that was a numerically established value of γ (see Ref.1). However, the slope read from Fig.2 is around $0.3{\tau_\eta}^{-1}$, which is substantially larger than the above value. This difference can be understood as follows.

The conventional method to lead to the smaller value, $0.16\tau_\eta$, is based on the frequently used assumption by Batchelor [2], that is, the stretching rate, γ, is equal to the arithmetic average of the stretching rates, $\gamma_e^{(i)}$, of many infinitesimal surface elements. Since the area A of a material surface is the sum of $\delta A^{(i)}$ of surface elements ($A = \sum_{i=1}^{I} \delta A^{(i)}$), γ can be expressed in terms of $\gamma_e^{(i)}$ and $\delta A^{(i)}$ as

$$\gamma(t) = \frac{\sum_{i=1}^{I} \gamma_e^{(i)}(t)\, \delta A^{(i)}(t)}{\sum_{i=1}^{I} \delta A^{(i)}(t)} = \frac{\langle\!\langle\, \gamma_e\, \delta A \,\rangle\!\rangle}{\langle\!\langle\, \delta A \,\rangle\!\rangle} \quad \left(\neq \langle\!\langle\, \gamma_e \,\rangle\!\rangle \right), \tag{3}$$

where $\langle\!\langle \cdot \rangle\!\rangle$ denotes the arithmetic average over the surface elements. This exact relation tells us that γ is not the arithmetic average of $\gamma_e^{(i)}$ with equi-weight, but the average with statistical weight proportional to $\delta A^{(i)}$. Note that if the correlation between the weight $\delta A^{(i)}$ and the stretching rate $\gamma_e^{(i)}$ decayed in time, γ would be equal to the arithmetic average. However, the correlation never vanish because of the explicit relationship,

$$\delta A^{(i)}(t) = \delta A^{(i)}(0) \exp\left[\int_0^t \mathrm{d}t'\, \gamma_e^{(i)}(t') \right], \tag{4}$$

between them. Details of this point are described in Ref.3 from the mathematical viewpoint that the stretching of material object is a multiplicative process.

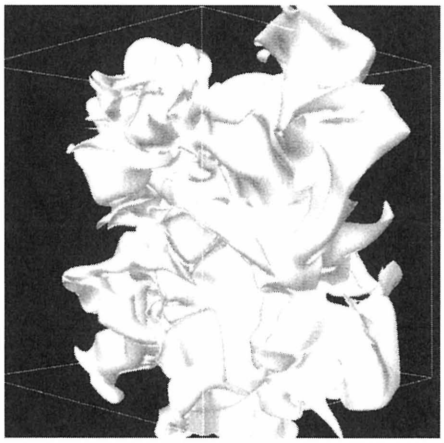

Fig.1 Deformed material surface in turbulence, which is flat initially. $R_\lambda = 57$. $t = 10\tau_\eta$.

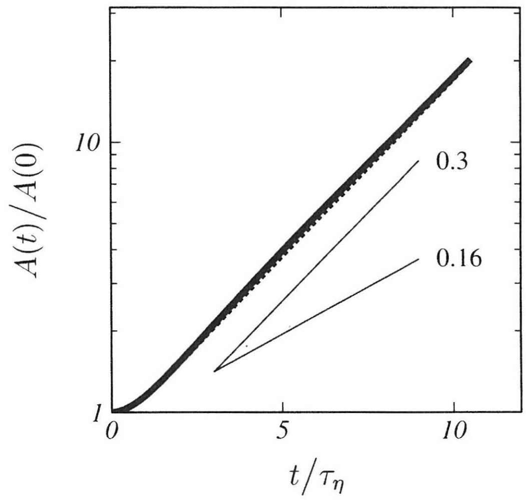

Fig.2 Exponential stretching of material surface area. Solid curve: $R_\lambda = 57$; dashed curve: 84. Two straight lines indicate slopes proportional to $\exp[0.3\, t/\tau_\eta]$ and $\exp[0.16\, t/\tau_\eta]$.

Reference

1) Girimaji, S. S. and Pope S. B., J. Fluid Mech. **220** (1990) 427.

2) Bathelor, G. K., Proc. Roy. Soc. London A **213** (1952) 349.

3) Goto, S. and Kida, S. J. Turbulence **3** (2002) 017.