§12. Neoclassical Transport Near the Magnetic Axis

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To consider neoclassical transport theory in the region near the magnetic axis in tokamaks, we must take into account the non-standard particle orbits appearing near the axis, of which widths are large. We study the application of Lagrangian formulation of transport theory[1,2,3], which has been developed to treat non-local nature of the neoclassical transport such as the effects of large banana width, to the transport near the magnetic axis.

Lagrangian formulation is constructed by transforming the independent variables of the Fokker-Planck equation in Cartesian coordinates $(\mathbf{x}, \mathbf{v})$ to three constants of motion (COM) (C_1, C_2, C_3) in axisymmetric systems. We choose the set of COM $(\mathcal{E}, \mu, \langle r \rangle)$, where $\mathcal{E}$ is the particle energy, μ is the magnetic moment, and $\langle r \rangle$ is the bounce-averaged radial position of a particle. In Lagrangian formulation with this set of COM, transport processes are described by the diffusion of average radial position of orbits by collisions. An advantage of this formulation is that the lowest order radial fluxes can be calculated from the local Maxwellian distribution function f_0 in the $(\mathcal{E}, \mu, \langle r \rangle)$ space, while we need to solve the perturbed distribution function f_1 to obtain fluxes in the conventional transport theory.

In the zero-banana-width approximation, which is valid away from the axis, neoclassical transport equations obtained by Lagrangian formulation agree with those obtained by the conventional Eulerian formulation. To apply this method to the region near the magnetic axis, and to develop the transport theory from previous works, we investigate these problems as follows:

1. Classification of particle orbits near the magnetic axis with COM $(\mathcal{E}, \mu, \langle r \rangle)$. We have developed the way how to classify the particle orbits sufficiently in the $(\mathcal{E}, \mu, \langle r \rangle)$ space[4].

2. The Jacobian in the $(\mathcal{E}, \mu, \langle r \rangle)$ space. We find the analytic expression of the Jacobian. It is proportional to the poloidal period of a particle motion determined by $(\mathcal{E}, \mu, \langle r \rangle)$.

3. How to treat collision terms. Because Fokker-Planck collision term is described with Euler variables $(\mathbf{x}, \mathbf{v})$, we need to transform it to the COM variables. An easy approximation is to treat $\langle r \rangle \simeq r$, which corresponds to the zero-banana-width approximation. However, since the typical banana width of particles passing near the magnetic axis becomes as large as $(q^2 \rho^2 R_0)^{1/3}$, this approximation is not correct. Another problem is that, because the collision terms are also averaged over particle orbits, we cannot use the conservation laws of collision terms, which are used in the usual Eulerian description of transport theory. These problems are now under investigation.

4. Interaction between electrons and ions. Previous works of Lagrangian formulation treat one-component plasma. To calculate transport coefficients more correctly, and to study the bootstrap current by Lagrangian formulation, we have to include the effect of unlike-species collisions. Because of the fact that the typical orbit width of ions is much larger than that of electrons, we cannot use the momentum balance between ions and electrons, which is not satisfied by the orbit-averaged collision term. Other properties which are often used in the usual Eulerian description of transport theory such as the ambipolarity of the radial fluxes and the quasi-neutral condition, should also be verified or modified in Lagrangian formulation. We are now constructing a proper approximation method to include electron-ion collision to the formulation.

5. The relation between the "orbit-averaged" fluxes in the $\langle r \rangle$ coordinate and those measured from experiments, or those calculated by the conventional Eulerian formulation, which gives the "flux-surface averaged" fluxes.

Our work will provide a new way to treat neoclassical transport near the magnetic axis, and it will also serve as a starting point for discussing the non-local treatment of neoclassical transport theory.

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