§11. Numerical Calculation of the Ion Thermal Conductivity by Lagrangian Neoclassical Transport Theory

Satake, S. (Grad. Univ. Advanced Studies),
Okamoto, M., Sugama, H.

To calculate neoclassical transport in the near-axis region, especially the ion thermal conductivity, we apply Lagrangian formulation[1]. The transport coefficients A_{jk} are obtained by taking moments of the reduced drift-kinetic equation in the constant-of-motion (COM) space. They are found to take the following form

$$A_{jk}(\langle\psi\rangle) = \sum_{\sigma_t} \int dx d\lambda_0 \tau_p \lambda_0 y(x) F_{jk}(x, \lambda_0, \langle\psi\rangle; \sigma_t), \quad (1)$$

where $x = \exp(-\mathcal{E}/T_i)$, $\lambda_0 = \mu B_0/\mathcal{E}$, τ_p is the poloidal period, and $y(x)$ is the Chandrasekhar function, respectively. Note that the summation is taken over all the types of orbits, which is labeled by σ_t, of which the averaged radial position are on the given $\langle\psi\rangle$. The integrands F_{jk} are functional of some orbit-averaged values, such as $\langle v_\parallel/B\rangle$, $\langle v_\parallel^2/B\rangle$, etc.

Numerical calculation of transport coefficients eq. (1) is implemented by using Monte Carlo integration method. In the calculation, test particles which have a given $\langle\psi\rangle$ are generated randomly and uniformly in the phase space $(x, \lambda_0; \sigma_t)$. And, all the functions in the integrand of A_{jk}, which we write $G_{jk}(x, \lambda_0, \langle r\rangle; \sigma_t)$ here, are calculated by tracing each particle orbit. Then, transport coefficients at $\langle\psi\rangle$ are given as

$$A_{jk}(\langle\psi\rangle) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} G_{jk}(x_n, \lambda_{0n}, \langle\psi\rangle; \sigma_{tn}), \quad (2)$$

where N is total number of test particles and $(x_n, \lambda_{0n}; \sigma_{tn})$ is the position of n-th test particle in the phase space.

In Eulerian transport theory, the ion heat flux is expressed as follows

$$\frac{q_i}{T_i} = -n_i \chi_i^r \frac{d}{dr} \ln T_i, \quad (3)$$

where χ_i^r is the ion thermal conductivity in the r direction. To compare our reslut with eq. (3), we rewrite the Lagrangian transport equation as

$$\frac{q_i^{\langle r\rangle}}{T_i} = -\bar{n}_i \chi_i^{\langle r\rangle} \frac{d}{d\langle r\rangle} \ln T_i, \quad (4)$$

where $chi_i^{\langle r\rangle}$ is proportional to A_{22}, and the radial coordinate is changed from $\langle\psi\rangle$ to $\langle r\rangle$. The ion heat conductivity $\chi_i^{\langle r\rangle}$ defined in this way is compared to χ_i^r.

As a example, we calculate the ion thermal conductivity $\chi_i^{\langle r\rangle}$ under the conditions $B_0 = 4$T, $q = 3$, $T_i = 20$keV and $\bar{n}_i = 1 \times 10^{20}\mathrm{m}^{-3}$. The radial electric field $d\Phi/dr$ is neglected. In this case, typical potato particles appear in the region $\langle r\rangle < r_p = 0.24$m.

The ion thermal conductivity is shown in Fig. 1. The dashed line is obtained from standard neoclassical theory[2]. A significant reduction of the thermal conductivity is seen in the near-axis region $\langle r\rangle < r_p$, where potato particles indeed dominate neoclassical transport.

In this region, the factor λ_q, which is introduced in the derivation of Lagrangian formulation, is still almost unity. It deviates from unity only in the region $\langle r\rangle < 3cm$. Then direct comparison of χ_i between Eulerian and Lagrangian theory is reasonable, and the reduction of χ_i in the core region of tokamak is expected. Our calculation reslts supports the recent results of both other Monte Carlo simulations and experiments, which suggest a significant reduction of χ_i from that obtained by standard neoclassical theory.

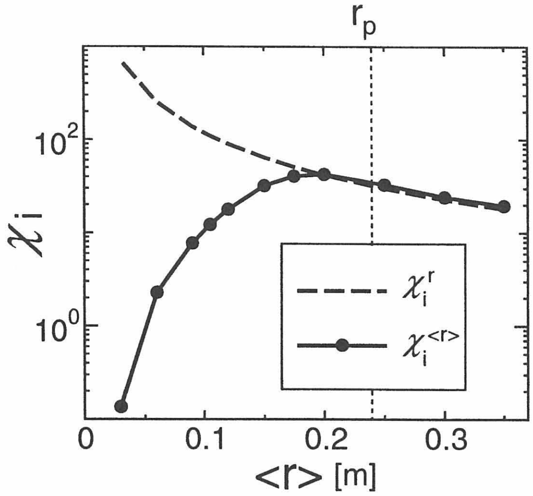

Fig. 1 : The ion thermal conductivity in the near-axis region.

1)Satake, S., Okamoto, M., Sugama, Phys. Plasmas (to be published).
2)F. L. Hinton and R. D. Hazeltine, Rev. Mod. Phys. **48**, 239 (1976).