§15. Nonlocal Stability Analysis of Microinstabilities in Inhomogeneous Plasmas

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In the present work, **analytical** methods such as WKB method associated with both differential equation and integral equation in wave number space (k-space) and numerical analysis with gyrokinetic integral equation are reviewed. Drift waves such as ion temperature gradient modes (SWITG) and electron temperature gradient modes (SWETG) in short wavelength region are studied with a gyrokinetic integration code in both slab and toroidal plasmas.

We here discuss the WKB analysis in different two approaches on the basis of the differential equation and the integral equation in k-space.

(1) WKB Solution of second order differential equation

For simplicity, we first consider an electrostatic modes in an inhomogeneous plasmas, which is described by the following equation,

$$\frac{d^2\phi}{dz^2} - Q(z)\phi = 0, \qquad (1)$$

where $\phi(z)$ is the electrostatic potential and $Q(z)$ is complex in general.

We finally obtain the WKB solution as

$$\phi(z_c) = \sum_{+,-} A_{\pm} Q^{-1/2}(z_c)\exp[\pm \int_{z_1,z_2}^{z_c} Q^{1/2}(z)dz]$$

and the eigenvalue relation can be derived from the connection formulae between the solutions of inside and outside of turning points as

$$\int_{z_1}^{z_2} [-Q(z_c)]^{1/2} dz_c = \pi(n+1/2), \; (n = 0,1,2,....)$$

(2) WKB Solution of Integral Equation in K-Space

We also discuss the highly localized electrostatic modes in an inhomogeneous (x-direction) plasma in case of $k\rho \geq 1$. The Vlasov – Poisson equations give the following integral equation in k-space instead of (1)

$$(k^2 + k_y{}^2 + k_z{}^2)\phi(k) = \int K(k',k;\omega)\phi(k')dk'$$

and we introduce an eikonal function in k-space in a form,

$$\phi(k) = \exp(-i\int g(k')dk'),$$

then, we finally have the following so called quantization condition,

$$\oint g_0(k)dk = 2(n+1/2)\pi$$

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