§4. An Equilibrium Equation of a Magnetized Plasma Including Electric Fields

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The equilibrium of tokamak plasma is sustained by a toroidal current and described by the Grad-Shafranov equation. On the other hand, Stix proposed the plasma confinement by a magnetoelectric torus where the poloidal plasma rotation reduces the charge accumulation. Based on these proposals, we here derive an equilibrium equation of a magnetized plasma including electric field. This equation represents the description of both tokamak and magnetoelectric torus mentioned above.

In order to get an equilibrium equation, we start from MHD equations. Momentum balance equation of steady state plasma is

$$nm(\vec{v}\cdot\nabla)\vec{v} = -mC_s^2\nabla n - \rho_c\nabla\phi + \vec{j}\times\vec{B}, \quad (1)$$

Here, $\vec{v}$ is the plasma flow velocity, n and m are the density and ion mass, respectively. Also, $\vec{j}$ the current density, $\vec{B}$ the magnetic field, ρ_c the electric charge density, ϕ is the plasma potential, respectively. Here, $C_s^2 \equiv (\kappa T_e + \gamma\kappa T_i)/m$. The Ohm's law in case of zero resistivity is

$$-\nabla\phi + \vec{v}\times\vec{B} = 0. \quad (2)$$

The electric charge density ρ_c is determined by Poisson's equation,

$$\nabla^2\phi = -\rho_c/\varepsilon_0. \quad (3)$$

The equation of continuity for the magnetic field, the electric current density and plasma flow density are $\nabla\cdot\vec{B}=0$, $\nabla\cdot\vec{j}=0$, $\nabla\cdot(n\vec{v})=0$. An axisymmetric plasma using cylindrical coordinates, r,ϑ,z is employed. Introducing the following flux functions, namely, magnetic flux function $\psi(r,z)$, poloidal current flux function $I(r,z)$, poloidal flow flux function $S(r,z)$ as

$$B = 1/2\pi r(\nabla\psi\times\hat{\vartheta} + \mu_0 I\hat{\vartheta}),$$

$$\vec{j} = 1/2\pi r(\nabla I\times\hat{\vartheta} - \hat{L}\psi\hat{\vartheta}/\mu_0),$$

$$n\vec{v} = 1/2\pi r\nabla S\times\hat{\vartheta} + nv_\vartheta\hat{\vartheta}.$$

Here, $\hat{L}\psi = r\partial/\partial r(\partial\psi/r\partial r) + \partial^2\psi/\partial z^2$.

Finally, we get the Grad-Shafranov equation including electric field in case of neglecting the convective derivative term, as

$$\frac{1}{(2\pi r)^2}\left[\frac{1}{\mu_0}\hat{L}\psi + \mu_0 I\frac{\partial I}{\partial\psi}\right] + mC_s^2\frac{\partial n}{\partial\psi}$$

$$-\varepsilon_0\Delta^2\phi\frac{\partial\phi}{\partial\psi} = 0. \quad (4)$$

If we neglect the potential term, this Equation reduces the Grad- Shafranov equation and becomes the equation derived by N. G. Popkov if magnetic surface ψ is neglected. Both tokamak and magnetoelectric torus equilibrium are involved in Eq. (4).