§9. Level-Crossing Function in the Analysis of Edge Plasma Turbulence

Rajković, M. (Vinča Institute of Nuclear Sciences, Belgrade, Serbia),
Watanabe, T.-H., Škorić, M.M.

Two important features of intermittency are its clustering property and the variability in amplitude. Different events cluster together creating uneven density in space and time, and events reflected in the highly variable amplitude are dispersed in space and time disproportionately. Much insight into the nature of intermittency may be gained from the study of approximations of turbulent signals which neglect amplitude aspect[1]. In this approach amplitude variations and local frequency of oscillations are separated by retaining only the zero-axis crossings (frequency) information (it is assumed here that the process under study has a mean value equal to zero). In a related approximation conveying somewhat different information about the original signal, turbulent signal is approximated by a binary approximation (BA) of the original signal and hence may assume values of 1 and 0. A general relation relating the slope σ_s of the complete neutral fluid turbulent signal and the slope σ_{BA} of its BA is proposed and heuristically proved based on the fractal (multifractal) property of the zero-crossing set on the line, namely

$$\sigma_{BA} = (1/2)(\sigma_s - 1) \qquad (1)$$

In Fig. 1 the spectra of 6861 L-mode of MAST and of its BA approximation are presented showing that relationship (1) is preserved so that this particular data set shares the spectral scaling properties with the neutral fluid turbulence.
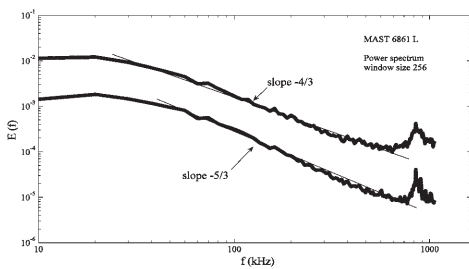

Fig. 1. Spectrum of the 6861 L-mode (slope=-5/3) and of its binary approximation (slope=-4/3)

Stochastic Catastrophe Theory and Plasma Turbulence

According to the stochastic variant of deterministic catastrophe theory (CT) the maximum likelihood estimation of stable and unstable equilibria are associated with the modes and anti-modes, respectively, of the system's stationary probability density function so that the deterministic potential function $V(x)$ and analogously the stationary probability density $P(x)$ provide the same information about the equilibrium points. Stable states may be considered as states of high probability while unstable

equilibrium states are states with low probability. The major advance in development of the invariant stochastic CT was the finding that the product of $P(x)$ and the diffusion function $I(x)=P(x)D(x)$, is invariant under coordinate transformation[2]. Noise is additive rather than multiplicative in the case of constant $D(x)$ and the pdf provides accurate information about configuration of equilibrium points since $P(x)$ is proportional to the generalized pdf $I(x)$. From the aspect of practical applications the crucial finding is that the generalized pdf I(x) can be evaluated from the level-crossing analysis of the stochastic signal under study[2]. The level-crossing function is a generalization of the probability density function and its multimodality indicates the number of equilibria of the system represented by the stochastic differential equation. In order to illustrate the accuracy of information provided by the kernel density estimate of the pdf and the level-crossing function, consider the L-mode and the H-mode turbulence data recorded in the MAST device. In Fig. 2 the two pdf's show no difference whil in Fig. 3 level-crossing function suggests two stable states while standard pdf one stable state. This supports the bifurcation scenario according to which the ELMs may be regarded as transition between different confinement regimes[3,4].
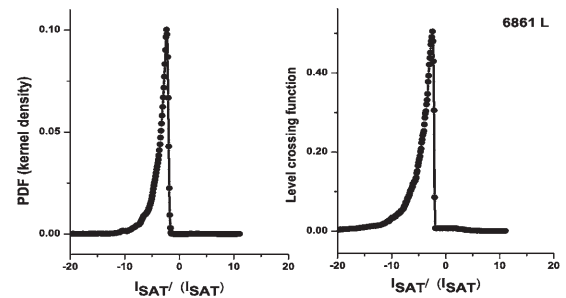

Fig.2 Kernel density estimation of pdf of 6861 L-mode (left) and the corresponding invariant level-crossing function (right).
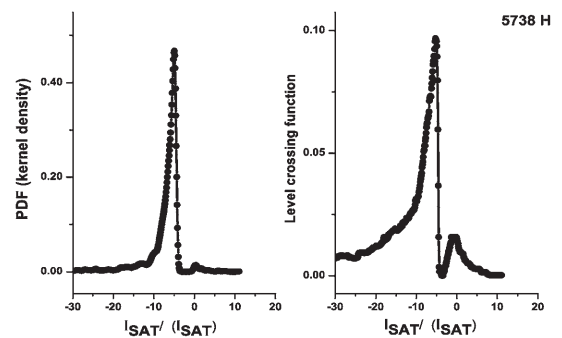

Fig. 3 Kernel density estimation of pdf of 5738 H-mode (left) and the corresponding invariant level-crossing function (right).

1) K. R. Sreenivasan et al., J. Stat. Phy., 125 (2006) 1145.
2) E-J. Wagenmakers, et al, Physica D 211 (2005) 263.
3) S-I Itoh et al., Phys. Rev. Lett. 76 (1996) 920-923.
4) M.R. Rajkovic et al., IAEA TM, Kyoto (2009).