§4. Phase Space Fluid Simulation of Ion Neoclassical Transport

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We have developed a new phase space fluid simulation code for investigation of ion neoclassical transport. Separating f into $f = f_0 + f_1$ according to small gyroradius ordering, we can write the drift kinetic equation as

$$\frac{\partial f_0}{\partial t} + \vec{v}_{\parallel} \cdot \nabla f_0 + \vec{a} \cdot \nabla_v f_0 - C(f_0, f_0) = 0 \quad (1)$$

and

$$\begin{aligned} \frac{\partial f_1}{\partial t} &+ (\vec{v}_{\parallel} + \vec{v}_d) \cdot \nabla f_1 + \vec{a} \cdot \nabla_v f_1 + \vec{v}_d \cdot \nabla f_0 \\ &- C(f_1, f_0) - C(f_0, f_1) - C(f_1, f_1) = 0. \quad (2) \end{aligned}$$

A steady state solution to Eq.(1) is a local Maxwellian distribution function

$$f_M = \frac{n_0}{\pi^{\frac{3}{2}} v_t^3} \exp\left(-x^2\right), \quad (3)$$

where $n_0(r)$ is the zeroth order density, r minor radius, $v_t = \sqrt{\frac{2T}{m}}$ the thermal velocity, $T(r)$ the zeroth order temperature and $x = \frac{v}{v_t}$.

In this work we calculate the collision as

$$\begin{aligned} C \simeq\ & C(f_1', f_M) \\ & - \frac{12\pi v_{\parallel} f_M}{(3\pi + 4) n_0 v_t^2} \left(erf(x) - \frac{2x \exp(-x^2)}{\sqrt{\pi}} \right) \\ & \times \int v_{\parallel} C(f_1', f_M) d^3 v \\ & - \frac{\sqrt{2\pi}}{n_0 v_t^2} \frac{f_M}{x} \left(erf(x) - \frac{4x \exp(-x^2)}{\sqrt{\pi}} \right) \\ & \times \int v^2 C(f_1', f_M) d^3 v, \quad (4) \end{aligned}$$

where we neglected second order terms. Thus to calculate the collision term, we first calculate the flow velocity and determine f_1' which in turn is used in Eq.(4). Equation (4) conserves a shifted Maxwellian distribution function almost completely with small numerical error in the calculation of the fluid velocity.

In order to verify that our numerical scheme works well for ion neoclassical transport calculation, we test our code for the following case. If we neglect $\vec{v}_d \cdot \nabla f_1$ term in Eq.(2), which is of second order but might be important with present day tokamak discharge parameters in order to take into account finite banana width effect,

$$\frac{\partial f_1}{\partial t} + \vec{v}_{\parallel} \cdot \nabla f_1 + \vec{a} \cdot \nabla_v f_1 + \vec{v}_d \cdot \nabla f_M - C(f_1) = 0. \quad (5)$$

In the special case of zero ion temperature gradient $\nabla T = 0$, this equation has the exact steady state solution of shifted Maxwellian[1]

$$f_{1t} = \frac{2 v_{\parallel} u_{\parallel}}{v_t^2} f_M \quad (6)$$

with the parallel fluid velocity

$$u_{\parallel} = \frac{q v_t^2}{2\epsilon\Omega} \frac{d \ln n_0}{dr}. \quad (7)$$

In Fig.1, we present the results of numerical calculation of Eq.(5) with zero temperature gradient. In Fig.1(a) and (b) we can confirm that the numerically calculated radial particle and energy fluxes indeed drop to zero at steady state. We also present the distribution function in Fig.1(c) and (d), where we can recognize that our numerical result agrees well with the analytical solution in Eq.(6) with slight difference in fluid velocity and thus, in the distribution function.

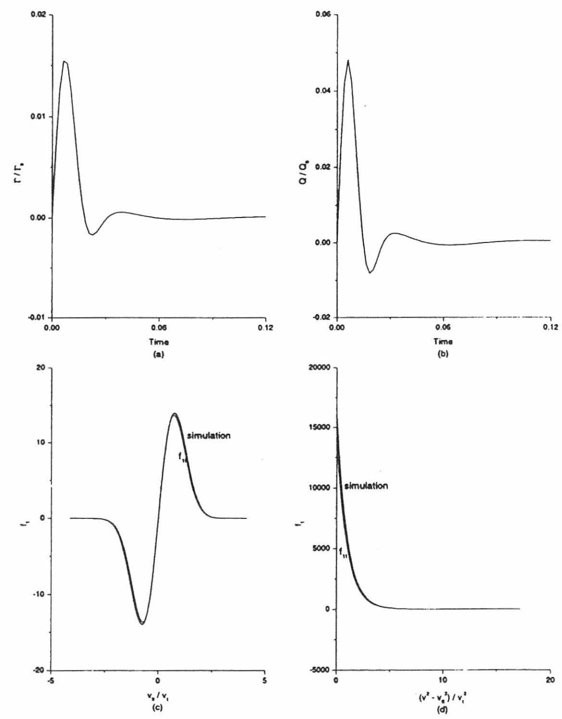

FIG. 1. Results of the numerical calculation of Eq.(6): (a) particle flux, (b) energy flux, (c) ion distribution function as a function of parallel velocity, and (d) ion distribution function as a function of squared perpendicular velocity.

References

[1] F.L. Hinton and R.D. Hazeltine, Rev. Modern Phys. **48**, 239(1976).