§59. Development of New Analytical Method for Energetic Particle Confinement Using CXNPA Diagnostics

Osakabe, M.

A new experimental technique to investigate the particle confinement of energetic ions during their slowing-down process is developed for charge exchange neutral particle diagnostics. The method is based on the maximum entropy and the maximum likelihood method and is used for the neutral beam short pulse experiment.

Figure 1 shows the concept of the analytical method on NPA-signals for an ideal NB-blip experiment. In this figure, the NB is injected instantly at $t=0$ and the waveform of the NPA at injection energy(E_0) can be expressed by the delta function ($\delta(t)$). Since the NB particles are ionized all over along their injection path and the NPA is line-integrated measurement, energetic particles produced by the beam experience various slowing-down times depending on their locations in a plasma before they are measured by the NPA. If we neglect the energy diffusion during the slowing-down process, the waveform by the energetic particle at the energy of E_i circulating on the location of ρ can be expressed as $w_i(\rho)\delta(t-\tau_i)$, where $\tau_i(\rho)$ expresses the characteristic time of energy slowing-down from the injection energy(E_0) to the energy of E_i, and $w_i(\rho)$ is a spatial distribution of the energetic particle along the NPA sight line. In usual plasma parameters, the energy slowing-down times monotonically increasing as it goes to the center from the edge. Therefore, if we looked at a certain energy channel of the NPA during the blip experiment, the temporal behavior of the signal at the channel simply expresses the spatial distribution of energetic particles on its sight line. The ratio $w_i(\rho)/w_0(\rho)$ gives us the information of confinement ratio for the energetic particle during its slowing-down process from the energy of E_0 to E_i.

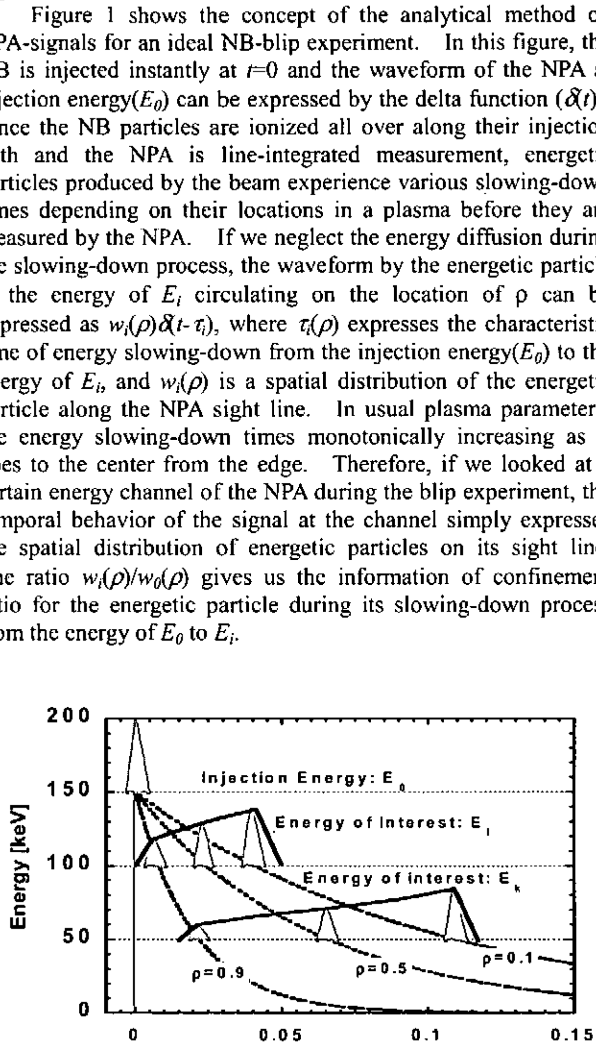

Fig.1 Schematic drawing of measured energetic neutral flux for the ideal NB-blip experiment. The thick-dashed lines express the energy slowing-down curves at various locations in the plasma. The triangle symbols express groups of energetic particles circulating around certain plasma locations with delta-function like waveforms. The thick-solid-line express the waveforms of the NPA at energies of interests (100-keV and 50-keV, in the figure).

In the actual case, the things are more complicated since the injection pulse width is finite. Suppose, the wave form of the NPA at the injection energy is expressed by $\psi_0(t)$. Introducing a response function $R_0(t)\equiv \psi_0(t)/\alpha$, the contribution of energetic particles on the location of ρ to the waveform of the NPA at the energy (E_i) can be expressed by $w_i(\rho)R_0(t-\tau_i)$ if we neglect the effect of energy diffusion in the slowing-down process. Here, the α is the normalized parameter so that it makes the integration of $R_0(t)$ over time to unity. The waveform of the neutral flux at E_i can be written as;

$$\psi_i(t)=\int_{\tau_{\min}}^{\tau_{\max}} w_i(\tau)R_0(t-\tau_i)d\tau \quad \text{---(1)},$$

where the τ_{min} and τ_{max} are the minimum and the maximum of τ_i on the NPA sight line. To evaluate the $w_i(\rho)$, we need to deconvolute the integral of Eq.(1). To treat the deconvolution numerically, we need to convert the waveform $\psi_i(t)$, $R_0(t-\tau)$, and $w_i(\rho(\tau))$ to series of discrete numbers. Dividing the region $[\tau_{min}, \tau_{max}]$ into n-regions, the Eq.(1) becomes;

$$\psi_i(t)=\sum_{j=1}^{n}\int_{\tau_j-\Delta\tau/2}^{\tau_j+\Delta\tau/2} w_i(\rho(\tau))R_0(t-\tau)d\tau$$

$$\cong \sum_{j=1}^{n} w_i(\rho(\tau_j))R_0(t-\tau_j)\Delta\tau=\sum_{j=1}^{n} w_{ij}R_j(t) \quad \text{---(1)'},$$

where $R_j(t)\equiv R_0(t-\tau_j)\Delta\tau$ and $w_{ij}\equiv w_i(\rho(\tau_j))$. Using the detection efficiency(η), the signal count(c_{ih}) by the NPA with in the time interval $[t_h-\Delta t/2, t_h+\Delta t/2]$ at the energy E_i can be written as;

$$c_{ih}=\eta\int_{t_h-\Delta t/2}^{t_h+\Delta t/2}\psi_i(t)dt=\sum_{j=1}^{n} w'_{ij}R_{jh}, \quad \text{---(2)}$$

where h is an index of a time bin of the NPA scaler, $w'_j\equiv \eta\, w_{ij}$ and $R_{jh}\equiv\int_{t_h-\Delta t/2}^{t_h+\Delta t/2}R_j(t)dt$. This equation can be written as $\vec{\mathbf{c}}_i=\mathbf{R}\cdot\vec{\mathbf{w}'_i}$ using a matrix expression. Since the actual detector count is fluctuating around the expected value $\overrightarrow{\langle \mathbf{c}_i\rangle}$ under the Poisson statistics, this equation is rewritten as;

$$\overrightarrow{\langle \mathbf{c}_i\rangle}=\mathbf{R}\cdot\overrightarrow{\mathbf{w}'_i}, \text{ or } \langle c_{ih}\rangle=\sum_{j=1}^{n} w'_{ij}R_{jh} \quad \text{---(2)'}.$$

We have to note that the **R** is an irregular matrix of n x m, where m denotes the number of the scaler time bins in the time range of our interest.

In the field of neutron spectroscopy, the problem like Eq.(2)' is known as 'an unfolding problem of pulse height spectra of scintillation detector'. Itoh had showed a numerical method of solving the problem like Eq(2)' using the maximum entropy method and maximum likelihood method under the Poisson statistics[1]. Applying his method to our problem, we obtain following equation;

$$p_{ij}\sum_{h=1}^{m} R_{jh}\left(c_{ih}/\langle c_{ij}\rangle-1\right)=0, \quad \text{---(3)}$$

where $\langle c_{ih}\rangle=(c_{tot}/Z)\sum_{j=1}^{n}R_{jh}\exp(y_{ij})$, $c_{tot}=\sum_{h=1}^{m}c_{ih}$, $p_{ij}=\exp(y_{ij})/Z$, $Z=\sum_{j=1}^{n}\exp(y_{ij})$, $w'_{ij}=c_{tot}p_{ij}$ and y_{ij} are fitting parameters. We will determine the value of y_{ij} numerically by using Newton's method, so that they will satisfy the Eq.(3). Detailed equations for numerical coding of the Newton's method and the error estimation are derived in Ref.1.

Reference

[1] S.Itoh *et al.*, J. Nucl. Sci. Technol. **26** (1989) 833