§1. Relation between the Radial Electric Field and the Flow in a Torus with $\nabla T = 0$

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The toroidal viscosity determines the neoclassical radial electric field in an axisymmetric tokamak plasma as in non-axisymmetric systems. This toroidal viscosity is very small and obtained only when the finite orbit width (FOW) effect is considered. In the case of uniform temperature ($\nabla T = 0$), the wave equation is derived to show the oscillatory behaviour of the radial electric field. The oscillatory radial electric field converges a steady state which satisfies the satandard neoclassical relation between the parallel flow and the radial electric field.

We consider only neoclassical transport to determine the radial electric field in axisymmetric tokamaks. It is well known, in axisymmetric tokamaks, that the particle transport is intrinsically ambipolar, which means that the electron and ion particle fluxes are independent of the radial electric field. To determin the radial electric field, it is nessecary to calculate the very small toroidal viscocity which is attributed to the FOW effect of particles.

We solve the drift kinetic equation including FOW effect [1].

$$\frac{\partial f}{\partial t} + \frac{e}{m}\frac{\partial \Phi}{\partial t}\frac{\partial f}{\partial \mathcal{E}} + (\vec{v_\parallel} + \vec{v_d}) \cdot \nabla f = C(f, f) \quad (1)$$

Only ions are treated in the present paper. This drift kinetic equation is solved numerically by the δf method with two weights [2].

In this paper, only the case of $\nabla T = 0$ is considered. For simplicity, it is assumed that the magnetic surfaces are concentric and circular. Then, we obtain

$$(1 + \frac{c^2}{v_A^2})\frac{\partial E_r}{\partial t} = -4\pi e \left\langle \int d^3 v f v_{dr} \right\rangle \quad (2)$$

The radial drift velocity is given by

$$v_{dr} = -\frac{v_\parallel^2 + v^2}{2\Omega_0 R_0} \sin\theta \quad (3)$$

$$v_{d\theta} = -\frac{E_r}{b} - \frac{v_\parallel^2 + v^2}{2\Omega_0 R_0} \cos\theta \quad (4)$$

where R_0 is the major radius and $\Omega_0 = eB_0/mc$ with B_0 the magnetic field at the axis. The parallel flow velocity has a form of

$$u_\parallel(r, \theta) = \langle u_\parallel \rangle \frac{B_0}{B} \quad (5)$$

with

$$B = B_0(1 - \frac{r}{R}\cos\theta) \quad (6)$$

Taking the time derivative of Eq.(2) yields

$$\varepsilon_\perp \frac{\partial^2 E_r}{\partial t^2} = -4\pi e \left\langle \int d^3 v \frac{\partial f}{\partial t} v_{dr} \right\rangle \quad (7)$$

The standard neoclassical theory gives an exact solution to the drift kinetic equation for the case of $\nabla T = 0$ [1]. The solution is a shifted-Maxwellian distribution function f_{SM}, which annihilates off the collision term ; $C = 0$. Inserting Eq.(1) into Eq.(7) yields

$$\frac{\partial^2 E_r}{\partial^2 t} + \omega_{GAM}^2 E_r = H \quad (8)$$

$$\omega_{GAM}^2 = \frac{7}{4}\frac{v_{th}^2}{R_0^2} \quad (9)$$

$$H = \omega_{GAM}^2 (\frac{T}{e}\frac{1}{n}\frac{dn}{dr} + \frac{rm\Omega}{eqR_0}\langle u_\parallel \rangle) \quad (10)$$

where ω_{GAM} is the GAM frequency. The wave equation is the same as that in Ref. [3] in the small $u_\parallel$ limit.

Equation (8) suggests that the radial electric field interacts with moving particles to generate an oscillation with a frequency ω_{GAM}. In the long time limit after the GAM oscillation damps out or after time averaging, the relation between the radial electric field and the flux averaged parallel flow becomes

$$\langle u_\parallel \rangle = -\frac{qR_0 v_{th}^2}{2r\Omega}(\frac{1}{n}\frac{dn}{dr} - \frac{e}{T}E_r) \quad (11)$$

This is just the relation given by the standard neoclaassical theory [1]. This suggests that the FOW effect may not affect the standard relation between the radial electric field and the parallel flow velocity.

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