§17. On the Two Weighting Scheme for δf Collisional Transport Simulation

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The validity is given to the newly proposed two weighting δf scheme in Ref.[1] for neoclassical transport calculations, which can solve the drift kinetic equation taking account of effects of steep plasma gradients, finite banana width, and the non-standard orbit topology near the axis.

Extending the phase space $(\vec{x}, \vec{v})$ to $(\vec{x}, \vec{v}, w)$ including the weight w, Chen and White obtained the weight equation as[2]

$$\dot{w} = \frac{1}{g}\left[-\int w S_M dw - \vec{v}_d \cdot \nabla f_0 + C(f_0, f_1)\right], \tag{1}$$

$$\frac{D}{Dt} g = \int S_M dw, \tag{2}$$

where S_M is the particle source, $\vec{v}_d$ is the drift velocity, and $C(f_0, f_1)$ is the collision operator for the background palsma. It is the key point to determine the marker distribution function g accurately. We apply the idea of δf method to solving equation (2) for g. The new weight equation for g contains another marker distribution function. We employ the δf method successively and obtain a hierarchy of weight equations, for $k = 1, 2, 3, \cdots$,

$$\dot{w}_k = \frac{1}{h^{(k)}}\left[-\vec{v}_d \cdot \nabla f_0 + \int \Omega_M^{(k-1)} dw_{k-1}\right], \tag{3}$$

$$\frac{D}{Dt} h^{(k)} = \int \Omega_M^{(k)} dw_k. \tag{4}$$

Since the source $\Omega_M^{(k)}$'s are arbitrary, we have taken

$$\int w_k \Omega_M^{(k)} dw_k = 0. \tag{5}$$

The marker density g_j for the $j-$th marker can be written as $g_j = g_{0j} + w_{1j} h_j^{(1)}$. Likewise, $h_j^{(k)} = h_{0j}^{(k)} + w_{k+1,j} h_j^{(k+1)}$. Under the assumption that $g_{0j} = h_{0j}^{(k)} = f_{0j}$ for all k, g_j can be written as

$$\begin{aligned} g_j &= f_{0j} + w_{1j}(f_{0j} + w_{2j} h_j^{(2)}) \\ &= (1 + w_{1j}) f_{0j} + w_{1j} w_{2j} (f_{0j} + w_{3j} h_j^{(3)}) \\ &= \cdots \\ &= (1 + w_{1j} + w_{1j} w_{2j} + w_{1j} w_{2j} w_{3j} + \cdots) f_{0j} \\ &\quad + w_{1j} w_{2j} w_{3j} \cdots h_j^{(\infty)}. \end{aligned} \tag{6}$$

We choose $\Omega_M^{(k)}$'s as, for $k = 1, 2, \cdots$,

$$\int \Omega_M^{(k)} dw_k = (1 + \epsilon_k) \int S_M dw. \tag{7}$$

Here, we will assume that $\epsilon_1 \neq \epsilon_2 \neq \cdots$ and $| \epsilon_k | \ll 1$ for all k. Then we can show that $w_k = w_1 + O(\epsilon_k)$ for $k = 2, 3, \cdots$.

The second term with $h_j^{(\infty)}$ on the right hand side in equation (6) vanishes because $| w_{kj} | < 1$ for all k. If we take an appropriate series for ϵ_k, equation (6) can converge to yield

$$g_j = (1 + w_{1j} + w_{1j}^2 + w_{1j}^3 \cdots) f_{0j} + O(\varepsilon) = \frac{f_{0j}}{1 - w_{1j}} + O(\varepsilon). \tag{8}$$

In the same way,

$$h_j^{(k)} = \frac{f_{0j}}{1 - w_{1j}} + O(\varepsilon). \tag{9}$$

In the lowest order (we can choose ε_k as small as possible), we recover the previous two weighting equations[1] ;

$$\dot{w}_j = \frac{1 - w_{1j}}{f_{0j}}\left[-\int w S_M dw - \vec{v}_d \cdot \nabla f_0 + C(f_0, f_1)\right]_j, \tag{10}$$

$$\dot{w}_{1j} = \frac{1 - w_{1j}}{f_{0j}}\left[-\vec{v}_d \cdot \nabla f_0 + \int S_M dw\right]_j. \tag{11}$$

Thus, the validity of the two weighting δf scheme proposed in Ref. [1] has been proved.

References

[1] Wang, W., Nakajima, N., Okamoto, M., Murakami, S., Plasma Phys. Contr. Fusion **41**, (1999) 1091

[2] Chen, Y., White, R. B., Phys. Plasmas **4**, (1997) 3591