§12. δf Simulation and the Radial Electric Field

Okamoto, M., Nakajima, N., Satake, S. (Grad. U. Advanced Studies),
Wang, W. (PPPL)

The satndard neoclassical theory does not hold near the magnetic axis since particle orbits near the axis are complicated and different from usual banana and passing particles [1]. The FOW (Finite Orbit Width) effect is important for such a plasma near the axis as well as for the plasma with steep pressure gradients. To take into account the effect of FOW, a δf Monte Carlo particle simulation code FORTEC [2] has been developed. The radial electric field has been calculated for the plasma with constant q and no toroidal rotation [3].

Here, we calculate the radial electric field in a rotated tokamak plasma by FORTEC. The rotation is modelled by a shifted Maxwellian velocity distribution.

The drift kinetic equation for ion distribution function is given, in $(\varepsilon, \mu, \vec{x})$ space, by

$$\frac{\partial f}{\partial t} + \frac{e}{m}\frac{\partial \Phi}{\partial t}\frac{\partial f}{\partial \varepsilon} + (\vec{v_\parallel} + \vec{v_{\rm d}}) \cdot \nabla f = C(f,f) \qquad (1)$$

The notations are standard. This drift kinetic equation is solved by the δf method which employs the two weighting scheme with accurate linear collision operator [2]. The shifted Maxwellian distribution function has a form of

$$f_{\rm SM} = e^{e\Phi/T}\frac{n}{(\pi v_{\rm th}^2)^{3/2}}e^{-m(\varepsilon - u_\parallel v_\parallel + u_\parallel^2/2)/T} \qquad (2)$$

Here, $u_\parallel$ is the shifted parallel velocity. It should be noted that $v_\parallel$ is a function of ε, μ, r, and θ.

The equation for the radial electric field becomes (cgs-unit)

$$\left(1 + \frac{c^2}{v_{\rm A}^2}\right)\frac{\partial E_{\rm r}}{\partial t} = -4\pi e\left\langle \int {\rm d}^3 v v_{\rm dr} f \right\rangle \qquad (3)$$

Here, $v_{\rm dr}$ is the radial drift velocity. Eq.(3) is solved with the time step less than the GAM (geodesic acoustic mode) period $\omega_{\rm GAM}^{-1}$ [4].

If we assume that the temperature is constant in space ($\nabla T = 0$), we can estimate the radial electric field by a simple analytical model neglecting the collision term $C(f,f)$. From Eqs.(1),(2), and (3), we obtain

$$\frac{\partial^2 E_{\rm r}}{\partial t^2} + \frac{e}{T}C_1 E_{\rm r} = C_2 \qquad (4)$$

The factors C_1 and C_2 can be easily calculated. In the limit of $t = \infty$ (after the GAM damps out), $E_{\rm r} = (T/e)(C_2/C_1)$ and we obtain

$$E_{\rm r} = \frac{T}{e}\left[\frac{\nabla n}{n} + 2\left(\frac{I_1}{I_0} - \bar{u}_\parallel \frac{d\bar{u}_\parallel}{dr}\right)\right] + \frac{T}{e}\alpha\frac{I_2}{I_0}\bar{u}_\parallel \qquad (5)$$

where $\alpha = 2r/(qR\rho)$, $\bar{u}_\parallel = u_\parallel/v_{\rm th}$, and I_0, I_1 and I_2 are integrals containing Gauss function to yield $I_1/I_0 \simeq 3.3\bar{u}_\parallel$ and $I_2/I_0 \simeq 0.43$.

In Fig.1, the calculation results are shown. Three cases of (1) $u_{\parallel 0} = 0$, (2) $u_{\parallel 0} = +0.1$, and (3) $u_{\parallel 0} = -0.1$ are illustrated. Simulation results (solid lines) and analytical estimations (dotted lines) are well in agreement. The agreement can be seen for the case of the plasma with steep pressure gradient. The comparison with an experimental measurement of $E_{\rm r}$ in a rotating plasma is under investigation.

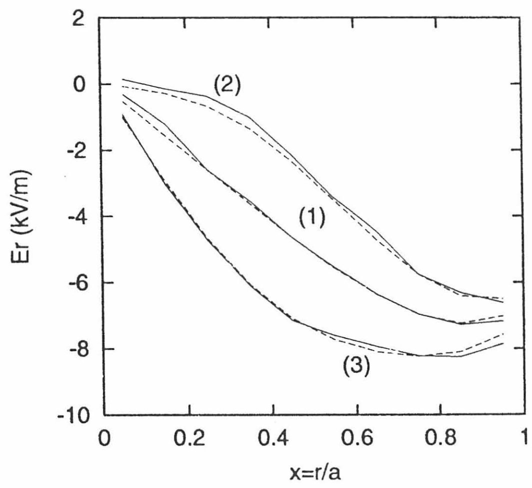

Fig. 1 Profiles in a rotating plasma.

References

[1] S. Satake, H. Sugama, M. Okamoto and M. Wakatani, J. Plasma Fus. Res. **77**, 573 (2001).

[2] W. X. Wang, N. Nakajima, M. Okamoto and S. Murakami, Plasma Phys. Control. Fusion **41**, 1091 (1999).

[3] W. X. Wang, F. L. Hinton and S. K. Wong, Phys. Rev. Let. **87** (5), 055002-1 (2001).

[4] N. Winsor, J. L. Jhonson and J. M. Dawson, Phys. Fluid **11** (11) , 2448 (1968).