§10. Particle Drift in Static Magnetic Fields

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1. Introduction

Analytical expressions for particle trajectory, and hence the drift velocity, in static magnetic fields are presented. The magnetic field configurations include: a slab configuration with a magnetic field gradient length L_B, and a toroidal configuration. These expressions for the particle drift velocity is valid irrespective of the ratio, $0<\rho/L_B<\infty$, where ρ stands for the gyro-radius. Also derived are analytical expressions for the guiding center motion on the drift surface in a toroidal plasma with a plasma current. In the case of the banana particles, the average excursion, Δr, from the magnetic surface is found to be positive. As a result, a banana particle makes the above radial displacement in a collision time τ.

2. Slab Configuration

Let us assume that the magnetic field has only z-component which varies along x: $B(x)=\left(1-x/L_B\right)B(0)$, where L_B denotes the length scale of magnetic field. Defining parameters

$$\varepsilon\equiv\frac{\rho_0}{L_B}=\frac{v_0}{\omega_0 L_B},\ \text{and}\ \ m\equiv\frac{4\varepsilon}{1+4\varepsilon\sin^2\left(\alpha/2\right)},$$

we have derived the exact drift velocity V_D normalized by the initial speed v_0 as follows:

$$\frac{V_D(m)}{v_0}=1-\frac{2}{m}\frac{K(m)-E(m)}{K(m)},\qquad(1)$$

where K and E are the complete elliptic functions of the first and the second kinds, respectively. The average x-position over the initial velocity direction, α, is

$$\langle x\rangle=x_0+\frac{\rho_0^2}{L_{\rm B}}\qquad(2)$$

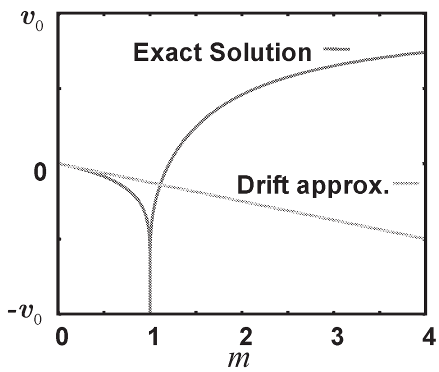

Fig. 1: Comparison of the exact ∇B drift velocity with drift approximation.

3. Toroidal Configuration

A banana particle moves around on a drift surface slightly deviated from the corresponding magnetic surface. Let us now assume that the circular magnetic surface on the poloidal plane (r,θ), and that two particles of the same species collide at a point $(r^+,\theta)=(r^-,\theta)=(r_0,0)$ at a time $t=0$. Other initial conditions are identical except for the pitch angle $\gamma_0^\pm=\gamma^\pm(0)$. Let the initial pitch angles are $\gamma_0^++\gamma_0^-=\pi$, i.e., the same parallel kinetic energy. Their radial positions of guiding centers are $r_{\rm G}^\pm(t)$. The conventional neoclassical theory, in which ρ^2 terms are ignored, gives the time average of $\overline{r}=r_t$ at the turning point, since $r(\theta)=r_t\pm\Delta r(\theta)$. If the plasma current profile $J(r)$ is either parabolic or filamentary, the average deviation $\overline{\Delta r}$ over the banana orbit can be shown to be

$$\overline{\Delta r}=\frac{4r_0}{\varepsilon_t}\left(\frac{q\rho}{a}\right)^2\left\{m-1+\frac{E(m)}{K(m)}\right\},\qquad(3)$$

where $m\equiv\cos^2\gamma_0/2\varepsilon_t$, and $\varepsilon_t\equiv r_t/R_0$ is the inverse aspect ratio with the major radius of R_0. The average of $\{\cdot\}$ (of the above equation) over the initial pitch angle γ_0 can only be obtained numerically, and is nearly $0.2\times\sqrt{\varepsilon_t}$. Thus, we have the average radial position and deviation:

$$\left\langle\overline{\Delta r}\right\rangle\approx0.80\frac{r_t}{\sqrt{\varepsilon_t}}\left(\frac{q\rho}{a}\right)^2\qquad(4)$$

In the case of $\gamma_0^\pm=90\pm25$ deg considered here, equation (3) gives $\overline{\Delta r}=4.0$ cm, while $\overline{\Delta r}=8.0$ cm numerically by using the Runge-Kutta method as shown in Fig. 2. The major radius is $R_0=3$ m. The particle energy is 1 MeV. A simple magnetic field configuration is adopted: the plasma current is concentrated on the magnetic axis at $r=0$. The toroidal field at the axis is 5 T and the plasma current is 2 MA. It should again be noted that the neoclassical theory gives $\overline{\Delta r}=0$ even though the collision is between unlike particles, whereas equations (3) and (4) in the present study deal with like particle interactions.

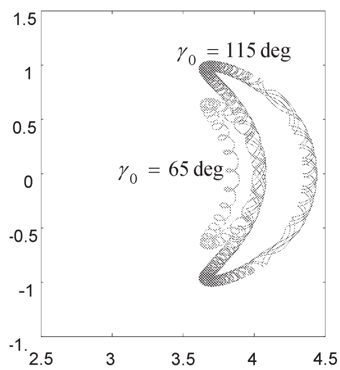

Fig. 2: Particle trajectories of the same species with the initial pitch angles of 65 deg and 115 deg at the starting point $(r_0,\theta_0)=(1,0)$ on the projected poloidal plane.