§5. Radiation of Hybrid Mode from a Circular Corrugated Waveguides

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Radiation of hybrid mode from a circular corrugated (CC) waveguide are closely related with the coupling of hybrid mode with Gauss-Laguerre beam at the mouth. By using the mode matching method, we analyze propagation of the multi-Gauss-Laguerre beams equivalent to the HE_{11} mode. We show evidence from the numerical calculations that radiation from two-, three- and multi-Gauss-Laguerre beams gives satisfactory approximation in both near- and far- field regions. The radiation pattern from multi-beam

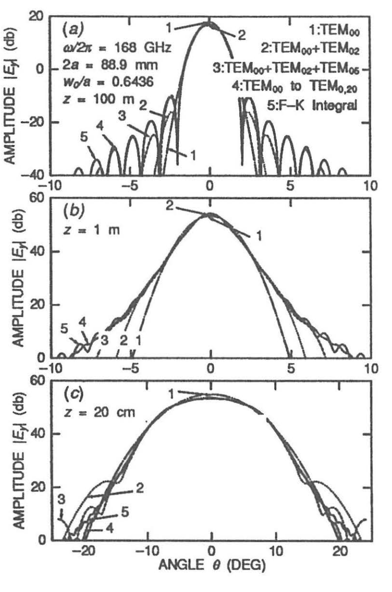

Figure 1: Radiation patterns of HE_{11} obtained from (1) the single TEM_{00} mode, (2) two-beam approximation, (3) three-beam approximation, (4) multi-beam approximation and (5) Fresnel-Kirchhoff integral for (a) z=100 m, (b) z=1 m and (c) z=0.2 m

is obtained from

$$E_y = \sum_{m=0}^{m_{max}} A_m E_{yTEM_{0m}} / \sqrt{\sum_{m=0}^{m_{max}} |A_m|^2}. \quad (1)$$

Because the orthonormality in the waveguide and free space is satisfied, the coefficient A_m can be calculated. In the calculation, 35 points of Gauss-quadrature integration with a relative incertitude of 10^{-9} is adopted. In two- and three-beam superpositions, the summation of ($m = 0,2$) and ($m = 0,2,5$) are carried out. In the LHD, the transmission lines which consists of the CC-waveguide with $2a$ = 88.9mm for $\omega/2\pi$=84 and 168 GHz are being prepared. In coupling between HE_{11} and TEM_{0m} for $w_0/a = 0.6436$, the coefficient A_m calculated. The TEM_{02} mode has the largest value in first negative coupling coefficient group and the TEM_{05} mode has the maximum value in second positive coupling group.

In Fig.1 (a)-(c), radiation patterns for different approximation at z = 100, 1 and 0.2 m are shown, respectively. With increasing superposition of higher modes in multi-beam approximation, radiation pattern becomes close to that of Fresnel-Kirchhoff (FK) integral. The main beam of FK integral can be approximated well with two-beam ($TEM_{00} + TEM_{02}$) or three-beam ($TEM_{00} + TEM_{02} + TEM_{05}$) superpositions in near- and far- field regions. In the multi-beam by sum of TEM_{0m} up-to $m = 20$, amplitudes of higher order side-lobes agree well with that of FK integral.

Amplitudes of side-lobe E_y for z = 100 m are examined as a function of m_{max}. Near a boundary value m between positive and negative coupling coefficients, a new side-lobe appears. With increasing m_{max}, the amplitude of the side-lobe is saturated. The amplitude of side-lobes in two- and three-beam approximations is nearly equal to that of the multi-beam approximation with $m_{max} = 2$ and $m_{max} = 5$. The deviation ϵ from the FK integral in single-, two-, three- and multi-beam superpositions with $w_0/a = 0.6436$ is calculated as a function of distance z, where the error ϵ is defined by

$$\epsilon = \frac{\int |\boldsymbol{E} \times \boldsymbol{H}^* - \boldsymbol{E}_{FK} \times \boldsymbol{H}^*_{FK}| 2\pi r dr}{\int \boldsymbol{E}_{FK} \times \boldsymbol{H}^*_{FK} 2\pi r dr} \quad (2)$$

For only far-field radiation, the single-beam approximation is slightly improved by changing $w_0/a = 0.596$ for 0.6436 as pointed out by Crenn [1]. In both near- and far-field radiations, two-beam and three-beam approximation is good with small ϵ. The multi-beam approximation such as m_{max} = 20 are best.

References

[1] Rebuffi, S. and Crenn, J.P., *International Journal of Infrared and Millimeter Waves*, **10**, 291-311, 1989.