§20. Tests of Analytical Formulas for the Parallel Viscosity in Drift-optimized Helical Configurations

Nishimura, S.,
Toda, A., Nakamura, Y. (Kyoto Univ.)

As reported previously [1], we had proposed two minor modifications for an analytical expression for the neoclassical parallel viscosity in the banana regime written in the Boozer coordinates [2]. One modification is reduction of non-axsymmetric ($n\neq 0$) Fourier components in the circulating particle distribution, and the other one is the elimination of a singularity in the Fourier expanded functions. Although these modifications retain the mathematical equivalence with the original formula before the modification, they provide numerous improvements in the approximation when using truncated Fourier series. The effects of the former modification can be clearly seen in so-called in drift-optimized configurations. Recent devices without quasi-symmetry often apply an idea of so-called σ-optimization to improve collisionless drift orbits. There are side-band helical and/or bumpy ripple spectra. Here we show a calculation example in such a configuration.

Figure 1 shows the magnetic field strength in a configuration with

$$B = B_0[1 - 0.1\cos\theta_B + 0.2\cos(\theta_B - 5\zeta_B) - 0.1\cos(5\zeta_B) - 0.1\cos(2\theta_B - 5\zeta_B)]$$

in the Boozer coordinates (θ_B, ζ_B), and B_0=1T, χ'=0.15T·m, ψ'=0.4T·m, B_θ=0, B_ζ =4T·m. The Fourier modes of (m,n)=(0,5) and (2,5) correspond to the side-band spectra. Figure 2 shows a comparison of three analytical formulas for the banana-regime parallel viscosity in this configuration. They are the Hamada coordinates version[3], the original Boozer coordinates version[2], and our improved formula for the Boozer coordinates[1]. Following Ref.[1], we show here the geometrical factor $G^{(BS)} \equiv -\langle B^2\rangle N^*/M^*$ instead of the mono-energetic viscosity coefficient N^* itself. All formulas are calculated using 88 Fourier modes for the circulating particle distribution function, and the technique to eliminate the singularity is applied for all of these formulas. As shown in the figure, the Hamada coordinates version and the improved Boozer coordinates version show good agreements with a numerical calculation using DKES (Drift Kinetic Equation Solver) code[1], but the original Boozer coordinates version deviates from them.

As discussed in Ref.[1], a main reason of this difference is that a original Boozer coordinates version derived by Shaing, Carreras, et al.[2] has a conflict with a basic idea in the previous theory developed in the Hamada coordinates [3]. A separation of two types distribution functions was used in the Hamada coordinates version. An important purpose of this separation was to separate effects of the high frequency modulation of the magnetic field strength modulation $\delta B/B$ (expressed by using notations H_1 and H_2 in Refs.[2-3]), which can be calculated without the Fourier expansion for the distribution, from a part requiring complicated calculations for the circulating particle distribution (written as $W(\lambda)$ in Refs.[1-3]). In spite of this original motivation, the coefficient $(\chi' m+\psi' n)/(\chi' m-\psi' n)$ in Ref.[2] emphasizes the high frequency modulation in many practical applications with $|\chi' m| << |\psi' n|$. The formula in Ref.[1] replacing this problematic coefficient by $(B_\zeta m+B_\theta n)/(\chi' m-\psi' n)$ with $|B_\zeta m| >> |B_\theta n|$ does not emphasize the high frequency modulation and contains them in a least necessary and sufficient level. This improvement of the numerical robustness will be important and useful in the recent drift-optimized helical configurations such as the inward shifted ones in the large helical device (LHD). The mathematical equivalence of original and improved versions of the formulas in the Boozer coordinates is discussed in another page of this report [4].

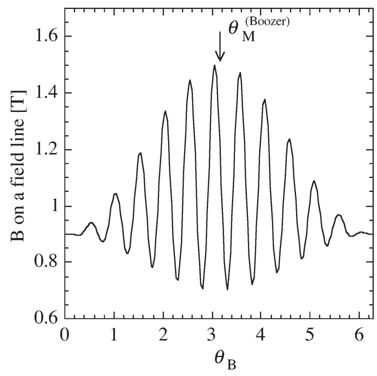

Fig.1 The model magnetic field used here.

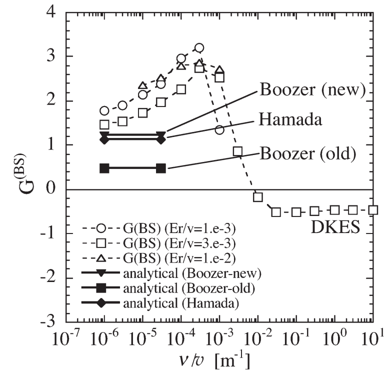

Fig.2 The geometrical factor expressing the parallel viscosity as the driving force for the neoclassical flows. The numerical results using the DKES (open symbols) and three asymptotic expressions (solid lines) are compared.

References

[1] Nishimura,S., Sugama,H., Nakamura,Y., Fusion Sci.Technol.51,61(2007); Ann.Rep.of NIFS Apr.2004-Mar.2005, p.327,p.328
[2] Shaing,K.C., et al., Phys.Fluids B1,1663(1989)
[3] Shaing,K.C., et al., Phys.Fluids B1,148(1989)