# §19. Development of a Moment Equation Solver for Multiple Ion Species Plasmas

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The calculation of neoclassical flows and the transport matrix is extended to arbitrary order of the Laguerre expansion (up to $j_{max}$=3) and to arbitrary number of particle species for future studies on bootstrap current, helium ash control, and impurity transport [1-2]. On the basis of a previous generalization (of the so-called Sugama-Nishimura method) to an arbitrary order of the expansion [3], the flux-surface-averaged parallel flow moments are determined by the parallel force balance equation

$$-\sum_{k=0}^{j_{max}} M^a_{j+1,k+1}\left\langle Bu_{\|ak}\right\rangle/\left\langle B^2\right\rangle + \sum_b \sum_{k=0}^{j_{max}} l^{ab}_{j+1,k+1}\left\langle Bu_{\|bk}\right\rangle .$$
$$= N^a_{j+1,1}X_{a1} - N^a_{j+1,2}X_{a2} + \delta_{j0} n_a e_a \left\langle BE_{\|}\right\rangle$$

Here, $M^a_{j+1,k+1}$ and $N^a_{j+1,k}$ are diagonal and non-diagonal parallel viscosity coefficients, respectively. Former coefficient describes the viscous damping of the flows and latter describes the driving force due to the radial gradient force $X_{a1}$ and $X_{a2}$. Non-diagonal coupling between arbitrary particle species (denoted by super- and sub-scripts $a$,$b$) caused by the friction collision coefficients $l^{ab}_{j+1,k+1}$ are fully included, and the parallel inductive electric field $\left\langle BE_{\|}\right\rangle$ also is retained for confirming the Onsager symmetry of the bootstrap current and Ware pinch coefficients. By solving this algebraic equation, the parallel flow moments $\left\langle Bu_{\|ak}\right\rangle$ are expressed as a linear combination of $X_{a1}$, $X_{a2}$, and $\left\langle BE_{\|}\right\rangle$. In this report, we show a numerical example using the mono-energetic DKES coefficients [4] in a LHD configuration ($R_{ax}$=3.6m, $B$=2.45T, $r/a$=0.5) [5] for obtaining the required viscosity coefficients $M^a_{j+1,k+1}$, $N^a_{j+1,k}$. Figure 1 shows a comparison of so-called 13M($j_{max}$=1), 21M($j_{max}$=2), and 29M($j_{max}$=3) approximations [3] for the bootstrap current and Ware pinch coefficients in a $e^-$+$H^+$+$Ne^{10+}$ plasma ($T_e$=2.0keV, $T_i$=1.0keV, $Z_{eff}$=5.74, $n_e$=1×10$^{18}$m$^{-3}$, $\partial\langle p_e\rangle/\partial r/\langle n_e\rangle$ = $\partial\langle p_i\rangle/\partial r/\langle n_i\rangle$ = $\partial\langle T_e\rangle/\partial r$ = $\partial\langle T_i\rangle/\partial r$ = −3.0keV/m) as functions of the radial electric field strength $E_r$.

The extended code has been used to calculate not only dependence of this kind of neoclassical quantities on the plasma parameters ($n_e$, $T_e$, $T_i$, $Z_{eff}$, $E_r$) but also their quality of the approximation by investigate the higher Laguerre order components in the distribution. Some other numerical examples as this convergence study on the Laguerre expansion in the LHD configurations are shown also in Refs.[1-2].

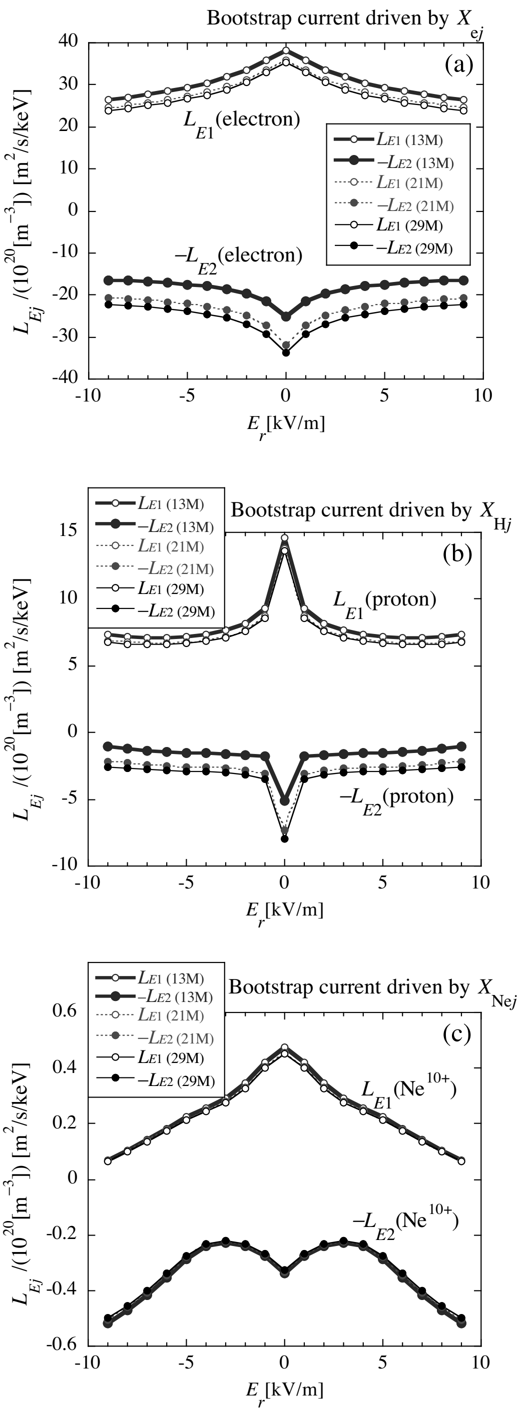


Fig.1 The bootstrap current and Ware pinch coefficients obtained by the 13M, 21M, and 29M approximations for a $e^-$+$H^+$+$Ne^{10+}$ plasma in a LHD configuration with $R_{ax}$=3.6m, $B$=2.45T (at $r/a$=0.5).

1) S.Nishimura, et al., in 17th ISHW and ICST (Princeton, 12-20 Oct. 2009) P02-16
2) S.Nishimura, et al., in this report
3) H.Sugama and S.Nishimura, Phys. Plasmas 15, 042502 (2008)
4) C.D. Beidler, et al., in 22nd IAEA FEC (Geneva, 13-18 Oct. 2008) TH/P8-10
5) S. Murakami, et al., Nucl. Fusion **42**, L19 (2002).