§21. On Analytical Expressions for Bootstrap Current Coefficients Applicable to Advanced Helical Devices

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A new connection formula for the geometrical factor associated to bootstrap currents is derived based on the drift kinetic equation divided into two parts: "equivalent symmetric" part and remaining "asymmetric" part. We consider at first the drift kinetic equation defined in Ref.[1] to derive the parallel viscosity driven by radial gradient forces,

$$V_{//}G_{Xa}-C_a^{L}(G_{Xa})=\sigma_{Xa} \quad (1),$$

where $V_{//}$ and C_a^{L} are the linearized Vlasov and linearized Coulomb collision operators, respectively. By applying a method used in the analytical theory for the banana regime [2-3], the source term σ_{Xa} corresponding to the radial gradient forces can be written in the form divided into "equivalent symmetric" and "asymmetric" contributions as

$$\sigma_{Xa}=\sigma_{Xa}^{(sym)}+\sigma_{Xa}^{(asym)} \quad (2)$$

$$\sigma_{Xa}^{(sym)}\equiv -\sigma_{Ua}\frac{c}{2e_a\chi'\psi}\left\{\frac{\psi' B_\zeta-\chi' B_\theta}{\langle B^2\rangle}+\frac{V'}{4\pi^2}H_2\right\} \quad (3)$$

Here, H_2 is a constant on the magnetic flux surface and is defined in Ref.[2]. Definitions of the other quantities in Eq.(2)-(3) are those in Ref.[1]. The source term $\sigma_{Ua}\equiv -m_aV_{//}(v\xi B)$ is that for the other drift kinetic equation $V_{//}G_{Ua}-C_a^{L}(G_{Ua})=\sigma_{Ua}$ to derive the flow-driven part of the perturbation. Then the solution of Eq.(1) is given by the linear combination of the solution of $V_{//}G_{Xa}^{(asym)}-C_a^{L}(G_{Xa}^{(asym)}) = \sigma_{Xa}^{(asym)}$ and G_{Ua}. The off-diagonal transport coefficient $N_a(K)$, which expresses the parallel viscosity due to the radial gradient forces, defined in Ref.[1] is obtained from the solutions. The analytical expression of the "asymmetric" part $N^{*(asym)}$ in the banana regime can be obtained by the procedure in Ref.[2]. The solving methods for the plateau and Pfirsch-Schlueter regimes are simple conventional ones. The remaining part of the coefficient $N^{*(sym)}$ due to the "symmetric" part of the perturbation G_{Ua} can be obtained directly from the analytical expressions for parallel viscosity coefficient M^* shown in Ref.[1]. Two individual connecting functions are used for these "symmetric" and "asymmetric" contributions in N^*. The most complicated part in these results is the banana regime term, which is already included in the mono-energetic geometrical factor in Ref.[4], in the asymmetric part. The other remaining parts are simple and thus the total computational effort for the formulas derived here is scarcely increased than that for the conventional formula.

Here, we assumed the magnetic configurations to be $B=B_0[1-\varepsilon_t\cos\theta_B+\varepsilon_h\cos(l\theta_B-n\zeta_B)]$, l=2, n=10, B_0=1T, χ'=0.15T• m, ψ'=0.4T• m, B_θ=0, B_ζ=4T• m. Figure 1 shows the results of the connection formula in a quasi-helical symmetric like configuration with ε_t=0.01 and ε_h =0.05. By adding the "equivalent symmetric" component (i.e. contribution of the helical ripple in this case) and the "asymmetric" component (that of the residual toroidicity) N^* having the polarity reversal at $\nu/v\sim 10^{-4}\mathrm{m}^{-1}$ is obtained. Figure 2 shows comparisons of geometrical factors $G^{(BS)}\equiv -\langle B^2\rangle N^*/M^*$ obtained by the connection formula derived here, with those obtained numerically by combining the DKES(Drift Kinetic Equation Solver) code[5] with our conversion formula $G^{(BS)} = -\langle B^2\rangle D^*_{13}/(D^*_{33}\nu/v)$[1]. For various non-symmetric configurations, the connection formula shows good agreements with the numerical results, especially in predicting the dependence of the polarity on the collision frequency.

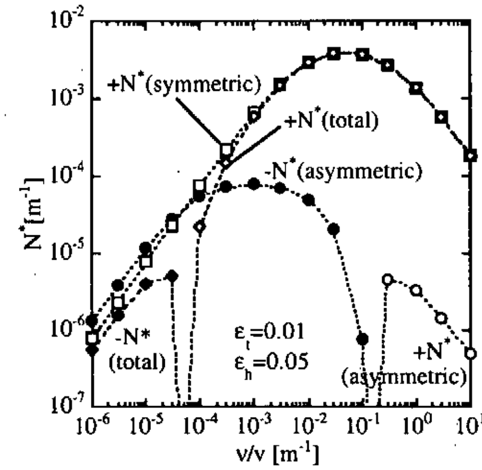

Fig.1 The connection formulas for the "symmetric"(squares) and "asymmetric"(circles) components of the mono-energetic coefficient N^*. The closed and open symbols indicate minus and plus values, respectively.

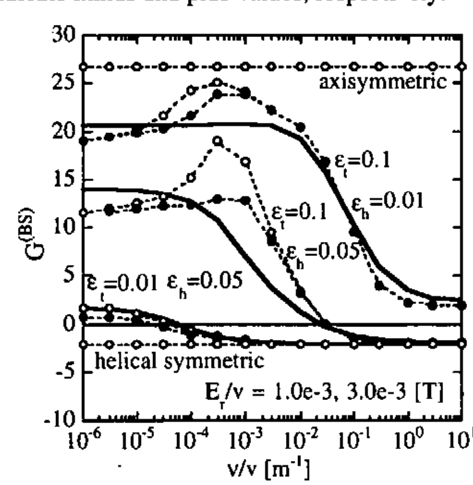

Fig.2 The geometrical factor $G^{(BS)}$. The connection formula (solid curves), and the numerical results for the cases with E_r/v=1×10^{-3}T(open circles) and 3×10^{-3}T (closed circles).

References

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