§28. Numerical Solutions of Modified K-dV Equation for Nonlinear Magnetosonic Wave

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It is well known that the nonlinear evolution of a magnetosonic wave in a plasma with one -ion species is described by the so called K-dV equation. For two ion plasma, however, the dispersion branch is split into two modes, namely,the high- and low- frequency modes. We note that the dispersion relation of this high frequency mode has a cut-off frequency. The linear dispersion relation under the influence of this finite cut-off frequency is described by

$$\omega \cong \mathrm{kV_A} - ak^3 + \frac{b}{k}.$$

Nonlinear evolution of this mode is described by a modified K-dV equation(M-KdV eq.)

$$\frac{\partial}{\partial\xi}\{\frac{\partial u}{\partial\tau} + \beta u\frac{\partial u}{\partial\xi} + \gamma\frac{\partial^3 u}{\partial\xi^3}\} - \delta u = 0.$$

It should be noted that the present M-KdV equation has only three conserved quantities although the conventional K-dV equation has infinite number of conserved ones. To study the general characteristics of solution of M-KdV equation , we solve numerically the equation by the 2 step Lax Wendroff method. We imposed the periodic boundary condition. As the initial condition, we discussed the following two cases:(1) a progressive pulse solution, $u_0 = A\sec h^2[..]$ (2) a periodic solution, $u_0 = A\sin[..]$. We choose the initial solution in the following numerical calculations as $u_{ini} = u_0 - \frac{1}{2L}\int_{-L}^{L} u_0 d\xi$.

A typical result of time evolution in case of (1) is shown in Fig.1. This result indicates a complicated nonlinear evolution process. Namely, an initial progressive soliton type solution propagates and gradually damps together with the excitation of short wavelength oscillations behind the main pulse. As time goes on, a new solitary type solution emerges from these short wavelength oscillations and this process occurs repeatedly. In case of (2), we observed so called recurrence phenomena of time evolution of the solution. It turns out that the recursion time becomes shorter due to the finite cut -off effect compared with the case of K-dV equation.

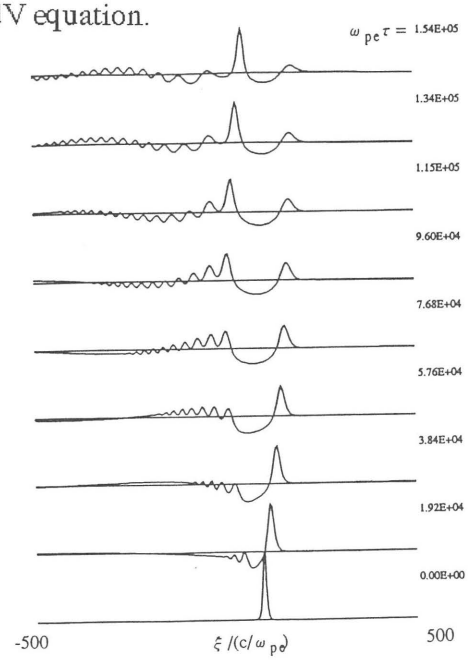

Fig.1 Time evolution of solitary pulse solution for a nonlinear magnetosonic wave