§4. Application of the CAS3D Code —Ballooning Modes in Tokamaks—

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The Finite-Element-Fourier (FE-Fourier) code package, CAS3D (Code for the Analysis of the MHD Stability of 3-D Equilibria),[1] which is based on a formulation of ideal MHD energy principle in the Boozer coordinates and developed by Nührenberg,C., provides the computational tool that is necessary to study the global MHD stability of 3-D toroidal plasmas. Within the framework of linear ideal MHD the energy integral for the plasma potential energy W_P connected with the displacement $\vec{\xi}$ can be given as

$$W_P = \frac{1}{2}\int d\vec{r}\left[|\vec{C}|^2 - \mathcal{A}(\vec{\xi}\cdot\nabla s)^2 + \gamma P(\nabla\cdot\xi)^2\right]$$

where s is the flux-surface label, and the destsabilizing term $\mathcal{A}$ and the stabilizing term $|\vec{C}^2|^2$ are expressed by

$$\begin{aligned}\mathcal{A} &= \frac{2}{|\nabla s|^4}(\vec{J}\times\nabla s)\cdot(\vec{B}\cdot\nabla)\nabla s,\\ \vec{C} &= \nabla\times(\vec{\xi}\times\vec{B}) + \frac{\vec{J}\times\nabla s}{|\nabla s|^2}\vec{\xi}\cdot\nabla s\end{aligned}$$

Since the fluid compressional contribution $\nabla\cdot\vec{\xi}$ has the stabilizing effects, the incompressible condition $\nabla\cdot\vec{\xi} = 0$ is used. Thus, only 2 scalar components, i.e., ξ^s and η of $\vec{\xi}$ are solved. To treat perturbations with a high toroidal mode number, the phase transformation is done as follows:

$$\begin{aligned}\xi^s &= X^e\cos[M\theta + N\phi] + X^o\sin[M\theta + N\phi],\\ \eta^s &= Y^e\sin[M\theta + N\phi] + Y^o\cos[M\theta + N\phi]\end{aligned}$$

where M and N are the poloidal and toroidal mode number, which is considered as the target mode number of the perturbation we treat. From this phase transformation, perturbations with a high toroidal mode number $N \gg 1$ are treated by not requiring much memory and CPU time.

Before applying the CAS3D code to heliotron/torsatron plasma so as to investigate low to high-n interchange and ballooning modes, it is applied to the ballooning calculation of a tokamak plasma as a bench mark test. The ballooning mode structure with $(M, N) = (7, -6)$ is expressed in Fig.1. And the ballooning mode structure with $(M, N) = (128, -96)$ is shown in Fig.2. Comparing these two figures, we can see the advantage of the CAS3D.

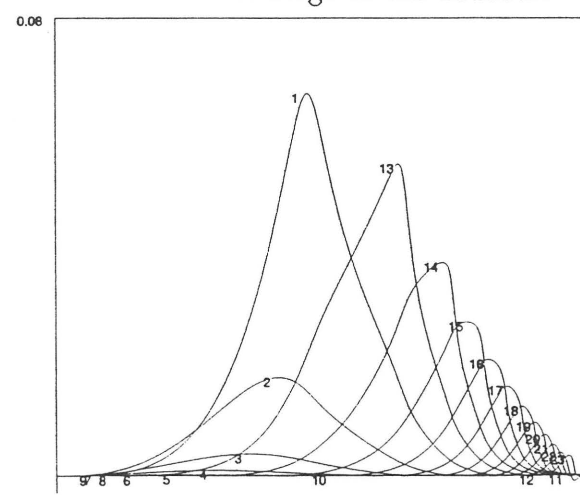

Fig.1 Ballooning mode structure of a tokamak plasma. The curve labeled 1 corresponds to the mode with $(M, N) = (7, -6)$.

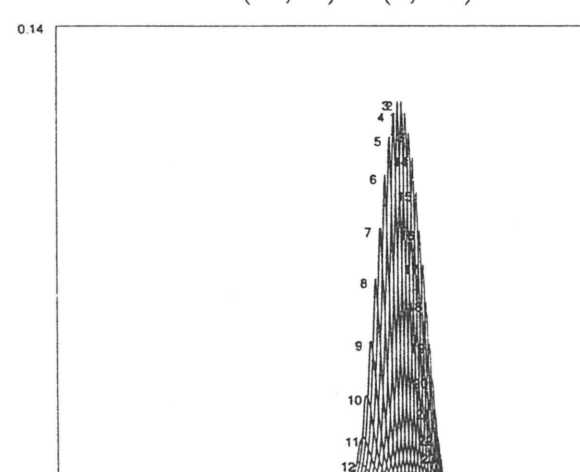

Fig.2 Ballooning mode structure of the same tokamak plasma. The curve labeled 1 corresponds to the mode with $(M, N) = (128, -96)$.

References
1) Schwab C. : Phys.Fluids B 5 (1993) 3195.