§7. Theoretical Analysis of Burst Modes in CHS

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Recently, fishborn-like burst modes (Burst modes) have been observed in $L = 2/M = 8$ Compact Helical System (CHS) experiments. In order to clarify the properties of such Burst modes theoretically, the formulation has been developed.

Since the Burst modes are considered to be driven by energetic fast ions, the treatment of fishborn instabilities in tokamaks [1] are extended. In the fishborn cases, $(m, n) = (1, 1)$ internal kink modes near the marginally stable state are destabilized by energetic fast ions. Thus, the dispersion relation is obtained by the variational method of the following functional:

$$D[\xi_r] = \delta W_F[\xi_r] + \delta W_I[\xi_r] + \delta W_K[\xi_r] \qquad (1)$$

where ξ_r is the radial displacement of $(m, n) = (1, 1)$ internal kink modes at the marginally stable state, and δW_F, δW_I, and δW_K are the potential energy of MHD fluid, the inertial energy, and the kinetic energy, respectively. Since such a ξ_r at the marginally stable state has discontinuity at the mode rational surface satisfying $\iota = n/m = 1$ (the approximate form is a step function), the inertial energy is introduced only vicinity of the mode rational surface, in order to construct the well-behaved continuous eigenfunction. The kinetic term is assumed not to contribute to determining the structure of the eigenfunction ξ_r, but to determining the eigenvalue ω through the dispersion relation. Thus, the functional given by Eq.(1) is rewritten as

$$\begin{aligned} D[\xi_r] &= \delta W_F^i[\xi_r] + \delta W_I^i[\xi_r] \\ &+ \delta W_F^e[\xi_r] + \delta W_K^e[\xi_r] \end{aligned} \qquad (2)$$

where superfixes s and e denote inertial layer near the mode rational surface and external region, respectively. This procedure are reasonable, because the driving source of $(m, n) = (1, 1)$ internal kink modes exist in the external region.

In the case of Burst modes in CHS experiments, MHD modes associated with them are considered to be ideal interchange modes. Since interchange modes have the tendency to be localized around their mode rational surfaces, the stability is determined only by the equilibrium quantities around their mode rational surfaces. This tendency becomes stronger near the marginally stable state or in the high-mode-number limit, and the stability is determined by the Mercier criterion (in three-dimensional systems) or the Suydam criterion (in one-dimensional systems) at their mode rational surfaces. Thus, for interchange mode, all the terms of the functional given by Eq.(1) should be considered simultaneously in the singular layer near their mode rational surfaces:

$$D[\xi_r] = \delta W_F^s[\xi_r] + \delta W_I^s[\xi_r] + \delta W_K^s[\xi_r]. \qquad (3)$$

On the basis of a stellarator-expansion, the linear growth rate of ideal interchange modes is obtained in Ref.[2] by using the asymptotic matching method of two solutions (one is a solution in the inertial layer, and the other is a solution outside of the inertial layer. Note that the singular layer consists of both regions). The linear growth rate is expressed in terms of the Suydam criterion.

On the basis of the method mentioned above, the dispersion relation is

$$\begin{aligned} &\sqrt{-\omega(\omega - \omega_{*i})} \\ &= 16r\frac{d\iota}{dr}\exp\left\{\frac{2}{u}\left[3\arg\Gamma\left(1+\frac{1}{2}iu\right)\right.\right. \\ &\quad -\arg\Gamma(1+iu) \\ &\quad \left.\left. -\tan^{-1}(\exp^{-\pi u/2}) - \frac{3}{4}\pi\right]\right\} \end{aligned} \qquad (4)$$

where $u = \sqrt{4(D_F + D_K(\omega)) - 1}$, and D_F and $D_K(\omega)$ are the Suydam criterion and a kinetic contribution, respectively.

References

[1] Chen. L and White. R. B, Phy. Rev. Lett., **52** (1984) 1122.
[2] Kulsrud. R. M, Phy. Fluids, **6** (1961) 904