§22. A Variational Analysis of Flow-reversal Condition in a Turbulent Swirling Pipe Flow Using the Bulk-helicity Concept

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The magnitude of axial-flow retardation near the center of a turbulent swirling flow is estimated from the results of the variational analysis with the aid of the helicity concept. It is analytically shown that the axial-flow reversal in a swirl occurs if the bulk helicity imparted to the mean flow exceeds the critical value, which is proportional to the square of the flux. It is suggested that the bulk helicity in the center region plays an important role in determining the flow-reversal condition. Through the comparison with the experimental observations in a turbulent swirling pipe flow, the reliability of the theoretically-derived reversal condition is confirmed [1].

One of the prominent features of the mean-velocity distributions in a turbulent swirling pipe flow, as compared with the counterparts in a usual turbulent (non-swirling) pipe flow, is a dip or dent of the axial velocity near the center of pipe. This is a persistent flow structure accompanied by a finite mean axial-velocity gradient. In order to investigate this feature of the swirling flow, we introduce two functionals that characterize the mean-flow structure in a turbulent swirling flow. Namely, the total amount of the mean-flow enstrophy Φ defined by

$$\Phi=\int_V W^2\, dV=\int_V (\nabla\times U)^2\, dV$$

and the total amount of the mean-flow helicity Ψ defined by

$$\Psi=\int_V U\cdot W\, dV=\int_V U\cdot(\nabla\times U)\, dV ,$$

where U is the mean velocity, $W(=\nabla\times U)$ is the mean vorticity, and V is the volume of the whole fluid region. The mean-flow helicity Ψ characterizes a swirling flow which is constituted by both the circumferential or azimuthal velocity and the axial or longitudinal velocity. This functional serves as a measure for the intensity of swirl.

We examine what velocity profiles are realized under a given intensity of swirl in the aggregate. To put it in terms of an extremalization problem, we seek a function U that extremalizes the functional Φ subject to a constraint that the total amount of the mean-flow helicity is constant: Following the usual procedure for a variation problem with a constraint, our conditioned variation problem is transformed into a free variation problem $\Phi+\lambda\Psi=extremum$, where λ is the Lagrange undetermined multiplier. We seek a flow distribution that satisfies a condition

$$\delta(\Phi+\lambda\Psi)=0 \qquad (1)$$

with respect to the velocity variation δU. This treatment is directly related to our considering the strong turbulence limit, where the transport coefficients such as λ are to be uniform in the whole region of the flow considered.

Equation (1) is solved, and the critical condition for the flow reversal is plotted on the plane of the flow helicity H per unit volume and total flux F is shown in Fig.1. Figures 2 shows the observation in experiments (by crossed-circle or by crossed-square) that all the swirling flows with reversal lie at the right of the corresponding H_c curve and that all those without reversal are at the left of the corresponding H_c curve. This comparison shows that this model explains essential element for the reversal in the swirling flow.

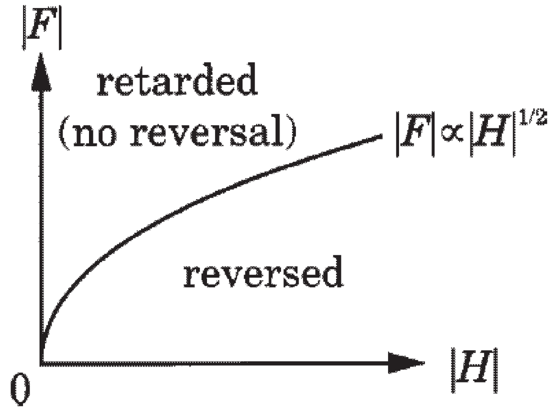

Fig.1 Critical condition for the reversal.

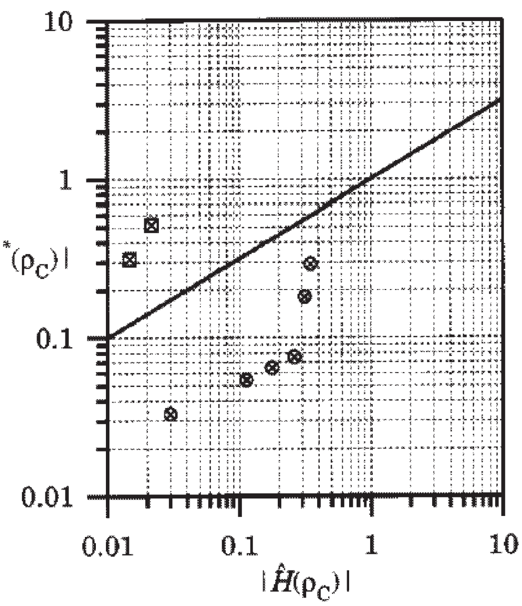

Fig.2 Experimentally observed data in swirling flows with an axial-flow reversal (circle) and that without (cross). The curves denote the critical bulk helicity.

Reference

[1] Yokoi N, Yoshizawa A, Itoh K, Itoh S-I, Phys. Fluids **16** (2004) 1186