§26. Orbit Mappings in Helical Systems

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Nonlinear behavior of magnetic field lines and drift orbits have been studies by a mapping method in helical systems. Nonlinear drift orbit mapping equation has been derived making use of the drift velocity: $v_\perp = v_E + v_d + v_D$ where v_E, v_d and $v_D = -D_e(1+\tau)\nabla n/n$ are ExB drift, diamagnetic drift , and dissipation drift velocities, respectively. The parallel compornent is given by the flow velocity along the magnetic field lines: $v_\parallel = v_\parallel B/B$. The effect of the dissipation induced from the resistivity in generalized Ohm's law is changed to a spatial diffusion coefficient D_e in the mapping equation. Integrating the drift velocities for one period of system length in the cylindrical coordinate system (r,θ,z), we obtain a mapping equation similar to the standard map:

$$x_{n+1} = x_n(1 - c_d) + K\sin(\ell\theta_n)$$

$$\theta_{n+1} = \theta_n + \iota(x_{n+1})$$

where x_n corresponds to the radial coordinate, c_d represents the effect of dissipation proportional to D_e, K represents the helical effect proportional to the helical magnetic field amplitude, ι is the rotational transform induced from the helmagnetic field, ℓ is the helical winding number, the effects of v_E and v_d have been neglected, and the density profile was assumed to be parabolic for the sake of simplicity[1].

Although our model may not be self consistent, it may be considered as a renormalization model similar to the rotational transform ι in a helical magnetic field, i.e., ι is produced by averaging a helical field which can be used again for the investigation of field line orbits.

The dissipation coefficient c_d is very small of the order of 10^{-6}~10^{-3} in actual confinement systems. Even these small dissipations, particle orbits change significantly. When the dissipation tends to zero, the mapping equation reduces to the area preserving one which is applied for magnetic field line orbits when drift effects are neglected. By varying the dissipation, therefore, we can study how the orbit configuration may change from the non-dissipative area preserving system to the dissipative system.

The dissipative orbit mapping points fill certain annular region with certain thickness almost ergodically, which looks stochastic, i.e. the orbit seems unstable. The stability of such orbit has been carefully examined by evaluating Fourier spectra and also the Lyapunov exponent. The frequency power spectrum calculated by the fast Fourier transform technique indicates that only a few peaks are observed instead of the broad noise spectrum. The two dimensional Lyapunov exponent is not positive. From these results we found that the dissipative orbit is not necessarily unstable even when the phase portrait of Poincare mapping looks stochastic.

The global stability characteristics of the helical configuration has also been examined by evaluating the Lyapunov exponent for 280x280 orbit starting points, which has been presented by two dimensional graphics. It is found that the dissipation does not change very much the global stability characteristics even for a large dissipation. An example is shown in Fig. 1, for the case of $\ell=1$ helical system, in which the dark region means the Lyapunov exponent is close to zero, i.e., stable region, while white region corresponds to largiest Lyapunov exponent or unstable region, and the gley region is intermediate unstable region. One also see the chaotic white regions around the separatrix and also self similar fractal structure.

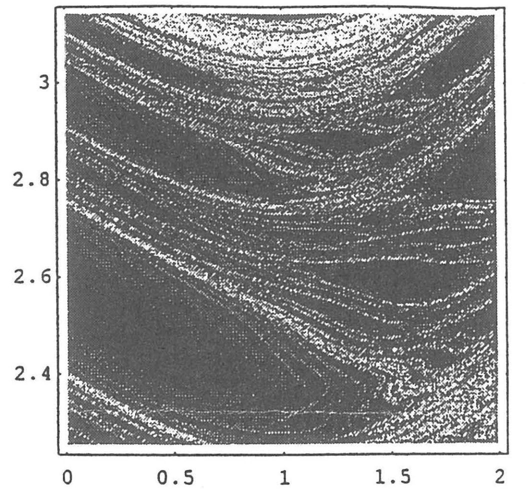

Fig.1

From these results, to examine the schocasticity of the dissipave orbits the Poincare mapping may not be suitable method. The Lyapunov exponent may be more reliable.

Frequency spectra for various orbits are also examined both in the $\ell=1$ and $\ell=2$ helical systems. The 1/f-type frequency spectrum is commonly observed in the $\ell=1$ configuration.

[1]T.Yamagishi, *Particle Orbit Mapping in Dissipative Helical Systems*, Report NIFS Proc-32, p.38-62 (1997).