§7. Use of DKES Code I: Accuracy Estimate

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The DKES code [1,2] can solve the linearized drift kinetic equation and give the mono-energy transport coefficients, without approximations on the collisionality. The mono-energy transport coefficients can be parameterized by ν/v and E_s/v, where ν is the energy-dependent collision frequency, E_s is radial electric field and v is the velocity. By taking v_{th} as the typical velocity, the normalized frequency $\nu(v_{th})/(v_{th}/qR)$ can be used to decompose the neoclassical transport into the different collisionality regimes. For example, the GSRAKE code [3] is valid for the $1/\nu$ regime. However, different species are possibly in the different regime, and also energy integral includes all the velocity ($v>>v_{th}$, $v<<v_{th}$). Thus how a code with approximations on the collisionality like the GSRAKE is good is practically unknown, until it is compared with a formal code on the collisionality like the DKES.

Thus I added service routines to the core sources of the DKES. Since it is needed to scan the electric field to impose the ambipolar condition, the interpolation of mono-energy transport coefficients will be practically needed on the parameters ν/v and E_s/v. Now only two species (e,i) are considered. The ratio of ν/v for the two species is at best of the order of 2 while the ratio of E_s/v is of the order of $(\text{mass ratio})^{1/2}$. Thus the mono-energy coefficients are calculated on 2 (not 4) planes with axes ν/v and E_s/v_j (j=e,i), and 2D interpolation is done on them for electron and ion separately.

As an example, the LHD magnetic field on a surface is considered (R=3.75m, ρ=0.6, $\nu/v_{th}\sim 0.65\varepsilon_{eff}^{3/2}$ thus in the $1/\nu$ regime). In Fig.(a), the interpolated mono-energy coefficients $D_{11}=(D_{11+}+D_{11-})/2$ (which is normalized by $(v/2)(Bv/\Omega)^2$) are plotted as a function of ν/v for some fixed E_s/v_j. Here D_-, D_+ are two approximated solutions corresponding to the extrema (minimum,max) for the mini-max variational principle [2]. For every points, the numbers of the Fourier and Legendre modes are $(m_{max},n_{max},l_{max})=(12,8,100)$ for the decomposition of the distribution function. The relative error defined by $E=2(D_--D_+)/(D_++D_-)$ is also plotted in Fig.(b). It seems that the error is too large (E~1) to use the DKES code for the small values of $\nu(v)/v$.

However, we can see that this bad convergence is not as bad as it seems. The small $\nu(v)/v$ needed for the energy integral is mainly due to the large v in the denominator ($\nu(v)$ also has v^{-4} dependence), i.e., the bad convergence comes from minority particles much faster than the v_{th}. For the Maxwellian equilibrium, the energy integral is weighted by the $\exp[-(v/v_{th})^2]$. Thus, to see the effective error, the error is multiplied by $\exp[-(v/v_{th})^2]$ in Fig.(c). It can be seen that maximum of the relative error is effectively reduced to less than 5% in this case.

The convergence will be improved by the increase of the mode numbers used. In Fig.(d), the relative error is plotted as a function of the total Fourier mode numbers ($\sim 2*m_{max}*n_{max}$), for the fixed maximum Legendre order (l_{max}=50,100,300) and fixed values of $(\nu/v, E_s/v)=(10^{-4}, -2*10^{-2})$. It can be seen that less than 100 of the Legendre order is enough for the convergence. On the other hand, the increase of the Fourier mode number will further improve the convergence. However the calculation time increases with the square of the Fourier mode number, while it is linear on the Legendre order. Thus it is important to use the Fourier modes by which the numerical accuracy is enough for the physical consideration and reasonable cpu time. An efficient matrix solver or simple parallelization will also be useful.

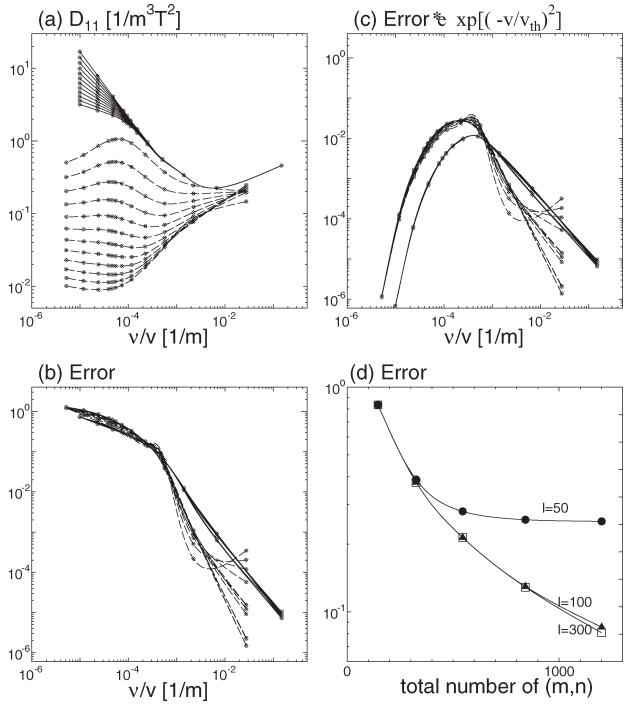

Figure: In (a)-(c), dashed and solid lines mean (i,e) respectively.

1) S.P.Hirshman et al., Phys. Fluids **29**, 2951 (1986)
2) W.I. van Rij et al., Phys. Fluids **3**, 563 (1989)
3) C.D.Beidler et al., Plasma Phys. Control. Fusion **37**, 463 (1995)