§10. Inclusion of Collision Term in Linear Gyrokinetic Eigenvalue Problem: I

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If collision term is relatively small, $|C(h_\sigma)/(\omega - \omega_d)h_\sigma| \ll 1$, it can be considered perturbatively in the linear gyrokinetic (GK) equation,

$$\frac{v_\parallel}{\mathcal{J}h}\frac{\partial h_\sigma^{(k)}}{\partial\theta} - i(\omega^{(k)} - \omega_d)h_\sigma^{(k)} = -i(\omega^{(k)} - \omega_*^T)\frac{eF_M}{T} \times (J_0\phi^{(k)} - v_\parallel J_0 A_\parallel^{(k)} - iv_\perp J_1 A_\perp^{(k)}) + C(h_\sigma^{(k-1)}).$$

Here k means kth calculation, and $k=0$ corresponds to collisionless case. Defining

$$\Psi_{0\sigma}^{(k)} \equiv J_0\phi^{(k)} - \sigma|v_\parallel|J_0 A_\parallel^{(k)} - iv_\perp J_1 A_\perp^{(k)},$$

$$C_\sigma^{(k)} \equiv C(h^{(k-1)})/\Big[-i(\omega^{(k-1)} - \omega_*^T)\frac{eF_M}{T}\Psi_{0\sigma}^{(k-1)}\Big],$$

and approximating the r.h.s of GK equation as

$$-i\frac{eF_M}{T}\Big[(\omega^{(k)} - \omega_*^T)\Psi_{0\sigma}^{(k)} + (\omega^{(k-1)} - \omega_*^T)\Psi_{0\sigma}^{(k-1)}C_\sigma^{(k)}\Big]$$

$$= -i\frac{eF_M}{T}(\omega^{(k)} - \omega_*^T)\Psi_{0\sigma}^{(k)}\Big[1 + \frac{\omega^{(k-1)} - \omega_*^T}{\omega^{(k)} - \omega_*^T}\frac{\Psi_{0\sigma}^{(k-1)}}{\Psi_{0\sigma}^{(k)}}C_\sigma^{(k)}\Big]$$

$$\simeq -i\frac{eF_M}{T}(\omega^{(k)} - \omega_*^T)\Psi_\sigma^{(k)},$$

with $\Psi_\sigma^{(k)} = \Psi_{0\sigma}^{(k)}(1 + C_\sigma^{(k)})$, then we have formal solutions

$$h_\sigma^{(k)} = \frac{eF_M}{T}(\omega^{(k)} - \omega_*^T)\frac{\mathcal{J}}{2\pi v}\int_{-\infty}^{\infty} dp\frac{|\omega_t|\exp[+i(p\hat\theta + \sigma w_d)]}{\omega^{(k)} - \sigma p|\omega_t|} \times \int_{-\infty}^{\infty} d\theta\frac{h}{|v_\parallel|/v}\exp[-i(p\hat\theta + \sigma w_d)]\Psi_\sigma^{(k)},$$

for circulating part with real number p, and

$$h_\sigma^{(k)} = \frac{eF_M}{T}(\omega^{(k)} - \omega_*^T)\frac{\mathcal{J}}{2\pi v}\sum_p\Big[\frac{\exp[+i(p\hat\theta + \sigma w_d)]}{\Gamma^{(k)} - \sigma p} \times \int_{\theta_1}^{\theta_2} d\theta\frac{h}{|v_\parallel|/v}\exp[-i(p\hat\theta + \sigma w_d)]\Psi_\sigma^{(k)} +\sigma(-1)^{p+1}\exp[+i\sigma(\Gamma^{(k)}\hat\theta + w_d)]\sum_{\sigma'=\pm}\Big(\frac{\sigma'}{\Gamma^{(k)} - \sigma' p} \times \int_{\theta_1}^{\theta_2} d\theta\frac{h}{|v_\parallel|/v}\exp[-i(p\hat\theta + \sigma' w_d)]\Psi_{\sigma'}^{(k)}\Big)\Big],$$

for trapped part with integer p. For definitions of $\omega_t, \Gamma, \hat\theta$ and w_d, see Ref.[1].

According to Ref.[1], Ritz method will be used to obtain a matrix eigenvalue problem, which require that integrals of the form of $\int d\theta(h/(|v_\parallel|/v))\exp[-i(p\hat\theta + \sigma w_d)]\Psi_\sigma$ in h_σ are linear combination of $\phi_l, A_{\parallel l}$ and $A_{\perp l}$. Here $(\phi_l, A_{\parallel l}, A_{\perp l})$ are expansion coefficients of $(\phi, A_\parallel, A_\perp)$ by basis function h_l, which is associated with lth Hermite polynomial [1]. In using Boozer coordinate, $(\phi, A_\parallel, A_\perp) = \sum_l(h_l/h)(\phi_l, A_{\parallel l}, A_{\perp l})$ is suitable. The $\Psi_{0\sigma}$ part in Ψ_σ is decomposed with h_l, by defining

$$\hat\Psi_{0\sigma l}^1 = \int_{\theta_{\min}}^{\theta_{\max}} d\theta\frac{h}{|v_\parallel|/v}\exp[-i(p\hat\theta + \sigma w_d)]\frac{J_0 h_l}{h},$$

$$\hat\Psi_{0\sigma l}^2 = \int_{\theta_{\min}}^{\theta_{\max}} d\theta\frac{h}{|v_\parallel|/v}\exp[-i(p\hat\theta + \sigma w_d)]\frac{\sigma|v_\parallel|J_0 h_l}{h},$$

$$\hat\Psi_{0\sigma l}^3 = \int_{\theta_{\min}}^{\theta_{\max}} d\theta\frac{h}{|v_\parallel|/v}\exp[-i(p\hat\theta + \sigma w_d)]\frac{iv_\perp J_1 h_l}{h},$$

where $(\theta_{\min}, \theta_{\max}) = (-\theta_M, \theta_M)$ for circulating particles with θ_M being for ∞ for numerical purpose, and (θ_1, θ_2) for trapped particles. The C_σ in Ψ_σ is decomposed as well,

$$\hat C_{\sigma l}^1 = \int_{\theta_{\min}}^{\theta_{\max}} d\theta\frac{h}{|v_\parallel|/v}\exp[-i(p\hat\theta + \sigma w_d)]\frac{J_0 h_l C_\sigma^{(k)}}{h},$$

$$\hat C_{\sigma l}^2 = \int_{\theta_{\min}}^{\theta_{\max}} d\theta\frac{h}{|v_\parallel|/v}\exp[-i(p\hat\theta + \sigma w_d)]\frac{\sigma|v_\parallel|J_0 h_l C_\sigma^{(k)}}{h},$$

$$\hat C_{\sigma l}^3 = \int_{\theta_{\min}}^{\theta_{\max}} d\theta\frac{h}{|v_\parallel|/v}\exp[-i(p\hat\theta + \sigma w_d)]\frac{iv_\perp J_1 h_l C_\sigma^{(k)}}{h},$$

Then the formal solution h_σ can be written as a linear combination such as $h_\sigma \propto \sum_{m=1}^3\sum_{l=0}^L(\hat\Psi_{\sigma l}^m + \hat C_{\sigma l}^m)\psi_l^m$, where $(\phi, A_\parallel, A_\perp) = (\psi^1, \psi^2, \psi^3)$ is defined. By appling $(T_e/e^2 n_e)\int d\theta h h_{l'}$ to the Poisson equation and the parallel and perpendicular Ampere's law including $(h_+ \pm h_-)$, we have a matrix with collisional effect for kth calculation. The calculation will be continued to obtain a convergent solution.

1) G.Rewoldt et al., Phys. Fluids **25**, 480 (1982)