§11. Inclusion of Collision Term in Linear Gyrokinetic Eigenvalue Problem: II

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If collision term is approximated as $C(h_\sigma) \simeq [C(h_\sigma^{(k-1)})/h_\sigma^{(k-1)}]h_\sigma^{(k)} \equiv -\nu_\sigma^{(k)} h_\sigma^{(k)}$, the linear gyrokinetic (GK) equation may be written as

$$\frac{v_\parallel}{\mathcal{J}h}\frac{\partial h_\sigma^{(k)}}{\partial\theta} - i(\omega^{(k)} + i\nu_\sigma^{(k)} - \omega_d)h_\sigma^{(k)} = -i(\omega^{(k)} - \omega_*^T)\frac{eF_M}{T}\Psi_{0\sigma}^{(k)},$$

where $\Psi_{0\sigma}^{(k)} \equiv J_0\phi^{(k)} - \sigma|v_\parallel|J_0A_\parallel^{(k)} - iv_\perp J_1 A_\perp^{(k)}$ is defined. Here k means kth calculation, and $k=0$ corresponds to collisionless case. In addition to $\hat\theta$ and w_d in Ref. [1], we may define for σ dependent complex function ν_σ,

$$v_\sigma(\theta) = \int_0^\theta d\theta' \frac{h\mathcal{J}}{|v_\parallel|}\nu_\sigma(\theta'),$$

for circulating part, and

$$v_\sigma(\theta) = \int_{\theta_1}^\theta d\theta' \frac{h\mathcal{J}}{|v_\parallel|}(\nu_\sigma(\theta') - \nu_\sigma^0),$$
$$\nu_\sigma^0 = \frac{1}{\tau_b}\int_{\theta_1}^{\theta_2} d\theta' \frac{h\mathcal{J}}{|v_\parallel|}\nu_\sigma(\theta'),$$
$$\Gamma_\sigma = (\omega - \omega_d^0 + i\nu_\sigma^0)/\omega_b,$$

for trapped part. Then we have formal solutions

$$h_\sigma^{(k)} = \frac{eF_M}{T}(\omega^{(k)} - \omega_*^T)\frac{\mathcal{J}}{2\pi v} \times \int_{-\infty}^{\infty} dp \frac{|\omega_t|}{\omega^{(k)} - \sigma p|\omega_t|}\exp[+i(p\hat\theta + \sigma(w_d + iv_\sigma))] \times \int_{-\infty}^{\infty} d\theta \frac{h}{|v_\parallel|/v}\exp[-i(p\hat\theta + \sigma(w_d + iv_\sigma))]\Psi_{0\sigma}^{(k)},$$

for circulating part with real number p, and

$$h_\sigma^{(k)} = \frac{eF_M}{T}(\omega^{(k)} - \omega_*^T)\frac{\mathcal{J}}{2\pi v}\sum_p \left[\frac{\exp[+i(p\hat\theta + \sigma(w_d + iv_\sigma))]}{\Gamma_\sigma^{(k)} - \sigma p} \times \int_{\theta_1}^{\theta_2} d\theta \frac{h}{|v_\parallel|/v}\exp[-i(p\hat\theta + \sigma(w_d + iv_\sigma))]\Psi_\sigma^{(k)} + \sigma(-1)^p 2i\sin(\Gamma_{-\sigma}\pi)\frac{e^{+i\pi\Gamma_+}\cdot e^{-i\pi\Gamma_-}}{e^{2i\pi\Gamma_+} - e^{-2i\pi\Gamma_-}}\sum_{\sigma'=\pm}\left(\frac{\sigma'}{\Gamma_\sigma^{(k)} - \sigma' p} \times \int_{\theta_1}^{\theta_2} d\theta \frac{h}{|v_\parallel|/v}\exp[-i(p\hat\theta + \sigma'(w_d + iv_{\sigma'}))]\Psi_{0\sigma'}^{(k)}\right)\right],$$

for trapped part with integer p. For definitions of $\omega_t, \omega_b, \hat\theta$ and w_d, see Ref.[1].

Now pitch angle scattering operator is considered as a specific collision model,

$$C = \frac{\nu_*}{2}\frac{\partial}{\partial x}(1-x^2)\frac{\partial}{\partial x},$$

with $x = v_\parallel/v$, and ν_* is usual energy dependent collision frequency. When distribution function f is decomposed by the normalized Legendre polynomial $p_l = \sqrt{l+1/2}P_l$ as $f = \sum_l \tilde f_l p_l$, the collision term is

$$C(f) = \frac{\nu_*}{2}\sum_l -l(l+1)p_l\tilde f_l,$$

where expansion coefficient $\tilde f_l$ is obtained as

$$\tilde f_l = \int_{-1}^1 dx p_l f = \sum_\sigma \int_0^1 |dx| p_l(\sigma|x|)f_\sigma(|x|) = \int d\Lambda \frac{1}{2h\sqrt{1-\Lambda/h}}\sum_\sigma p_l(\sigma|x|)f_\sigma(|x|),$$

if $\Lambda = h(1-x^2)$ is used as the pitch angle variable. The integral range is taken for both circulating and trapped particles. The Legendre function is oscillating with l and the summation with respect to l is usually much smaller than each l term. Thus it is necessary to estimate $\tilde f_l$ accurately in each θ mesh.

The pitch angle collision operator satisfies number and energy conservation automatically. Then momentum conservation will be imposed, $\sum_s(\sum_{s'}\int d^3v m_s\boldsymbol{v}C_{ss'}) = 0$. For arbitray weight W, the velocity integral of a function g becomes zero by changing g to g^*

$$g^* = g - F_M\left[\frac{\int d^3vgW}{\int d^3vF_MW}\right].$$

Here it is assumed that the field part of collision term is propotional to the Maxwellian. As an example, the parallel momentum conservation is considered. Then $g_s = C(h_s) = -\nu_s h_s$ and $W = (m_s v_{\parallel s} + (g_{s'}/g_s)m_{s'}v_{\parallel s'})$ for each s species, to obtain new ν_s function with parallel momentum conservation.

1) G.Rewoldt et al., Phys. Fluids **25**, 480 (1982)