§2.   Two Propagation Bands in Geodesic Acoustic Mode Oscillation

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In the previous section, unified response function is obtained eliminating so called "Constant Velocity Approximation (CVA)". In this section this model is numerically realized applying it to a simple tokamak case. Here, an approximation is employed for the non-uniform field in order to keep insights into physics during numerical calculations.

$$\tilde{\phi}_{l \neq 0}^{(n)} = -\frac{\sum_{l'} \sum_{1 \leq ns \leq n} \tilde{D}_{ns,l,l'}}{\tilde{D}_{\|,l}^{appro}} \tilde{\phi}_{l'}^{(n-ns)} \quad (1)$$

where

$$\tilde{D}_{\|,l}^{appro} \equiv -\sum_{s} (k_{D,s}^2 / k_{D,i}^2) L_{\tilde{\mu}}^{pas \sin g} \cdot \frac{1}{2} \xi_0(\tilde{\mu})(1 + \zeta_l Z_2(\zeta_l))$$

(2)

$$\xi_0(\tilde{\mu}) = (1/2\pi) \oint \tilde{g} / |\tilde{\upsilon}_{\|}| d\theta \quad \text{and} \quad g = \partial l / \partial \theta .$$

We used $\varepsilon = 0.2$ and $q = 3$ in the following calculations. In Eq. (2), trapped particles do not participate the parallel charge screening. This makes $D_{\|,l}^{approx}$ smaller than that obtained with CVA as shown in Fig.1. This enhances the non-uniform potential field, which in turn enhances the contribution to (uniform) induced charge. The response of the plasma to the non-uniform potential filed is shown in Fig. 2; note that separate treatment of passing and trapped particles is facilitated due to the elimination of CVA.   Fig. 3 shows the calculated fourth order induced charge separated into parts contributed by non-uniform fields $\phi^{(0)} \sim \phi^{(3)}$. Each term consisted of those due to considerably spread poloidal mode numbers. Enhanced neoclassical mode coupling is another results of eliminating CVA. Combining the 2nd and 4th order terms we obtain quadratic dispersion relation for Geodesic Acoustic Mode Oscillation. As shown in Fig. 4, the GAM frequency is now

a function of perpendicular wave number $k_{\perp}^2$ revealing 2 bands of propagation; the frequency that has been referred to as GAM frequency is identified to be the cutoff frequency in terms of the quadratic dispersion function.
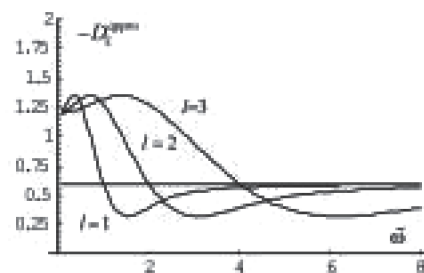

Fig.1 $\tilde{D}_{\|,l}^{appro}$ versus   normalized   frequency $\tilde{\omega} = \omega / (\upsilon_T / qR)$.
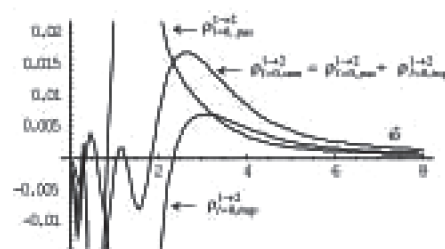

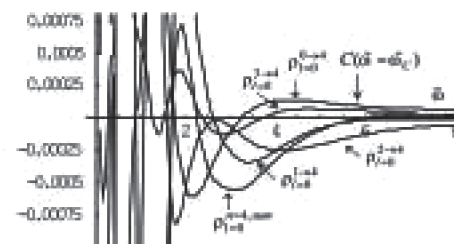Fig.2 The induced charge due to non-uniform field of various order of non-uniform field.

Fig.3 The 4-th order terms in smallness parameter $\varepsilon_p = (ik_{\psi} I \upsilon_T / \omega_c)$.

Fig.4 Propagation bands of GAM oscillation
Normalized perpendicular wave number
$\tilde{k}_{\perp}^2 = (k_{\perp} \upsilon_T / \omega_c)^2$ is taken for the coordinate.