## §1. Unified Linear Response Function of Zonal Flow with full Finite Orbit Effects

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A new formulation of the linear response function of electrostatic potential to nonlinear drive (due to turbulence) is derived in this section; zonal flows play important roles in the self-regulation of turbulence and their basic physics are contained in response functions. Two branches of zonal flows (stationary- zonal flow and Geodesic Acoustic Mode) are known to exist in the low and high frequency ranges. However, they have been analyzed separately using different approximations due the difference in their frequency ranges.

This paper visits this problem and gives a unified expression of the response function by taking full account of finite orbit effects. The drift kinetic equation is integrated along particle orbits by expanding them in Fourier series. Thus a separate handling of passing and trapped particles is facilitated revealing some important aspects of zonal flows:

$$\delta n_{ind} = (\tilde{g}/\tilde{B})\int d^3\upsilon f_{ind} = \frac{e\phi_{l'}}{T}\frac{1}{2}\sum_j \sum_n (i)^n \left(\varepsilon_p^n / n!\right) L_{\tilde{\mu}} \cdot \sum_{l''}\sum_m A_{0,n,m}^j \cdot B_{n,l'}^j \cdot (\alpha_{n-m_1,m,l''}^{j,c} + \alpha_{n-m_1,m,l''}^{j,s}) e^{i(l'+l'')\theta}$$

(1)

where

$$A_{0,n,m}^j \equiv \frac{B_0}{B_{\max}} \sum_{m_1,\sigma} \varepsilon_p^n \frac{1}{n!} \tilde{I}_{m_1,m}^j C_{n,m_1} (-1)^{n-m_1} , \quad (2)$$

$$\tilde{I}_{m_1,m}^{pas\sin g} \equiv \frac{1}{\tilde{T}_b}\int_{-\pi}^{\pi} \frac{\tilde{g}d\theta}{|\tilde{\upsilon}_{||}|}\left(\frac{\tilde{\upsilon}_{||}(\theta)}{\tilde{\omega}_c(\theta)}\right)^{m_1} \exp(im\tilde{\omega}_b u(\theta))$$

$$\tilde{I}_{m_1,m}^{trapped} \equiv \frac{1}{\tilde{T}_b}(1+(-1)^{m_1+m}) \times \int_{-\theta_{\max}}^{\theta_{\max}} \frac{\tilde{g}d\theta}{|\tilde{\upsilon}_{||}|}\left(\frac{\tilde{\upsilon}_{||}(\theta)}{\tilde{\omega}_c(\theta)}\right)^{m_1} \exp(im\tilde{\omega}_b u(\theta)) \quad , (3)$$

$$\alpha_{n,m_1,m,l'}^{c,j} = \frac{1}{2\pi}\int_{-\theta_{\max,j}}^{\theta_{\max,j}} \frac{\tilde{g}}{|\tilde{\upsilon}_{||}|}\left(\frac{\tilde{\upsilon}_{||}(\theta)}{\tilde{\omega}_c(\theta)}\right)^{n-m_1} \cos(m\tilde{\omega}_b u(\theta))\cos(l'\theta)d\theta$$

$$\alpha_{n,m_1,m,l'}^{s,j} = \frac{1}{2\pi}\int_{-\theta_{\max,j}}^{\theta_{\max,j}} \frac{\tilde{g}}{|\tilde{\upsilon}_{||}|}\left(\frac{\tilde{\upsilon}_{||}(\theta)}{\tilde{\omega}_c(\theta)}\right)^{n-m_1} \sin(m\tilde{\omega}_b u(\theta))\sin(l'\theta)d\theta$$

$$\alpha_{n,m_1,m,l'}^{j} \equiv \alpha_{n,m_1,m,l'}^{c,j} + \alpha_{n,m_1,m,l'}^{s,j} \quad (4)$$

and

$$B_{n,l'}^{trapped} = (-\zeta_{-m} Z_{n+2}(\zeta_{-m})) \qquad B_{l',m}^{pas\sin g} = (-\zeta_{l'-m} Z_{n+2}(\zeta_{l'-m})) \quad (5)$$

$$Z_{n+2}(\zeta_{l'-m}) \equiv 2\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} dx x^{n+2} e^{-x^2}\left(\frac{1}{x-\zeta_{l'-m}}\right). \quad (6)$$

In these equations j denote kind of trajectories, passing or trapped. $\theta_{\max} = \pi$ for passing particles and $\theta_{\max} = \theta_t$, the upper turning point, for trapped particles.

Using these neoclassical terms and other terms of Finite Larmor Radius Effect, in charge neutrality condition, the following linked equations are obtained determining response function:

$$k_{D,i}^2 \phi_{l=0} \sum_n \varepsilon_p^{(n)} \sum_{l',ns\le n} (\tilde{D}_{ns,l',-l'}\tilde{\phi}_{l'}^{(n-ns)}) + \rho_{ext}(\omega,k_\psi) = 0 \quad (7)$$

and

$$\sum_{l',ns\le n} (\tilde{D}_{ns,l',l-l'}\tilde{\phi}_{l'}^{(n-ns)}) e^{il\theta} = 0 \quad (8)$$

Here,

$$\tilde{D}_{ns,l',l''} = \sum_s \sum_j (k_{D,s}^2/k_{D,i}^2) L_{\tilde{\mu}}^j \cdot (-\frac{1}{2}\xi_{l-l'}^j(\tilde{\mu})\delta_{ns,0} + (\varepsilon/q)^{ns}(-\frac{1}{2}\tilde{\mu}\lambda_{l''}^{j,(n=2)}(\tilde{\mu})\delta_{ns,2} + \frac{1}{4!}\frac{9}{4}\tilde{\mu}\lambda_{l''}^{j,(n=4)}(\tilde{\mu})\delta_{ns,4}) + \frac{1}{2}(i)^{ns}(1/(ns)!)\sum_m \sum_{0\le m_1\le ns} (-1)^{n-m_1} \cdot I_{m_1,m}\alpha_{ns,m_1,m,l''}^j B_{ns,l',m}^j)$$

(9)

By using in Eq. (7), the solution to homogeneous Eq. (8)

$$\tilde{\phi}_{l'\neq 0}^{(n')} = a_{l'}^{n'}(\omega)\tilde{\phi}_{l=0} = a_{l'}^{n'}(\omega) \quad (10)$$

, we obtain the unified response function in the most general form.

$$\phi_{l=0} = -\frac{\rho_{ext}}{k_{D,i}^2 \sum_n \varepsilon_p^n (\tilde{D}_{n,0,0} + \sum_{1\le ns\le n} \tilde{D}_{ns,l',-l'} a_{l'}^{n-ns}(\tilde{\omega}))} \quad (11)$$