§5. New Theoretical Approach to 3D Equilibrium of the LHD

Watanabe, T., Hojo, H. (Plasma Research Center, Univ. Tsukuba)

LHD has potentiality of the high beta plasma confinement.

The chaotic field line-layer, which surrounds the outside of the outermost magnetic surface, plays a key role for an efficient plasma confinement in LHD. High magnetic shear of the LHD lengthens the connection length of the chaotic field lines. The connection length of the diverter field line which approaches close to the outermost magnetic surface exceeds 10 km. The cold plasma on surface of diverter plates does not cool down the core plasma directly therefore. Besides, the chaotic field lines neutralize the charge separation that cause the plasma collapse by an interchange mode. The lines of force that break away the chaotic field line region reach the vacuum vessel wall soon. Then, it is, also, expected in chaotic field line region that the plasma pressure can be sustained stably by the line-tying effect of the field lines fastened to the vacuum vessel wall.

The bootstrap current driven by the plasma pressure play another important role for the high beta plasma confinement. The bootstrap current reduce the magnetic field in the outside region of helical coils. Then, the bootstrap current contributes to the high beta stability of LHD plasma through the reduction of the total magnetic field energy.

The new approach to 3D equilibrium of the LHD does not depend on the assumption of the nested flux functions. The new approach is composed of the new expression scheme for the magnetic field and the introduction of a rotating helical coordinate system (X, Y, ϕ).

The magnetic field $\boldsymbol{B}$ satisfies always the relation $\nabla \cdot \boldsymbol{B} = 0$. Consequently, the essential freedom of $\boldsymbol{B}$ is two, and $\boldsymbol{B}$ can be expressed as follows by the 2 component of a vector potential $\boldsymbol{A}$ without loss of generality.

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}, \, \boldsymbol{A} = \begin{pmatrix} 0 \\ \Phi \\ p\{\Psi - X\Phi\}/r \end{pmatrix}. \qquad (1)$$

LHD equilibrium, $\{P(X, Y, \phi), \Phi(X, Y, \phi), \Psi(X, Y, \phi)\}$,

composed of magnetic surface region and chaotic field line region can be obtained numerically by the force balance equation

$$\nabla P = \boldsymbol{J} \times \boldsymbol{B}, \quad \boldsymbol{J} = \frac{1}{\mu_0} \nabla \times \boldsymbol{B}. \qquad (2)$$

A numerical example of $\{\Phi(X, Y, \phi), \Psi(X, Y, \phi)\}$ for the vacuum magnetic field of the LHD is shown in Fig.1.
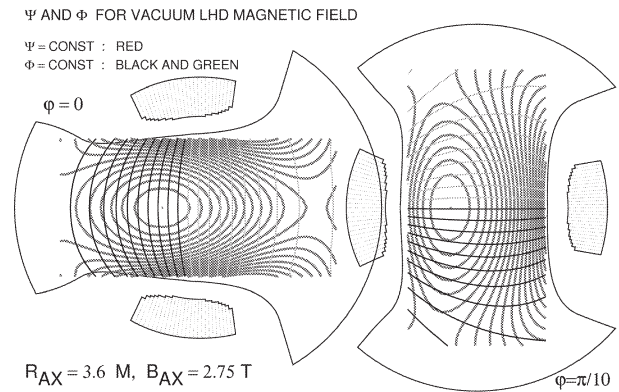

Fig.1 Numerical example of $\Phi(X, Y, \phi), \Psi(X, Y, \phi)$.

The MHD equilibrium (2) can be reduced to a compact differential equation for the Ψ, if analysis is constrained to the flux function region and introduced smallness parameters of $1/p$ (p is pitch parameter of helical coils $(= 5)$) and aspect ratio.

$$0 = \frac{r^2 + p^2 Y^2}{S} \frac{\partial^2 \Psi}{\partial X^2} + \frac{r^2 + p^2 X^2}{S} \frac{\partial^2 \Psi}{\partial Y^2} - \frac{2XY}{S} p^2 \frac{\partial^2 \Psi}{\partial X \partial Y}$$
$$- \frac{3r^2 + p^2(X^2 + Y^2)}{S^2} p^2 \left(Y \frac{\partial \Psi}{\partial Y} + X \frac{\partial \Psi}{\partial X} \right)$$
$$+ \frac{r^2}{S} F(\Psi) F'(\Psi) - 2p \frac{r^3}{S^2} F(\Psi) + \mu_0 \frac{r^2}{p^2} P'(\Psi) \qquad (3)$$

where $S \equiv r^2 + p^2(X^2 + Y^2)$ and $F(\Psi) = \frac{r}{p} B_\phi + X B_Y - Y B_X$ is an arbitrary function of Ψ. A numerical example of eq.(3) is shown in Fig.2.
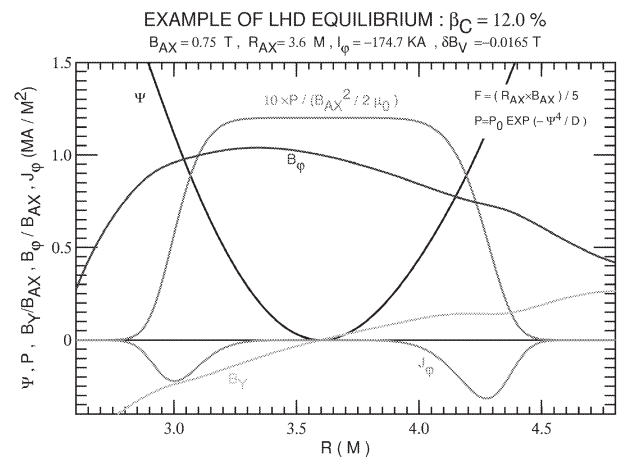

Fig.2 Numerical example of eq.(3) for the LHD