§36. A New Algorithm for Differential-Algebraic Equations Based on HIDM

Watanabe, T., Gnudi, G.

There are many excellent algorithms to solve the initial value problems described by non-stiff ordinary differential equations. We can usually get good solutions for such problems by excellent ready-made computer programs. However, we encounter sometimes serious numerical difficulties if the problems are reduced to stiff ordinary differential equations, or to differential-algebraic equations. Differential-algebraic equations frequently arise in many physical problems, such as optimal control problems, dynamical systems with constrained conditions and so on.

A new algorithm is proposed to solve differential-algebraic equations. The algorithm is an extension of the algorithm of general purpose **HIDM** (higher order implicit difference method). A computer program named **HDMTDV** and based on the new algorithm is constructed and its high performance is proved numerically through several numerical computations, including index-2 problem of differential-algebraic equations and connected rigid pendulum equations.

The new algorithm is also secular error free when applied to dissipationless dynamical systems. The new code can solve the initial value problem

$$\mathbf{0} = \boldsymbol{L}\left(\varphi(x), \frac{d\,\varphi(x)}{d\,x}, \frac{d^2\,\varphi(x)}{d\,x^2}, x\right) ,$$

where $\boldsymbol{L}$ and φ are vectors of length N. The values of first or second derivatives of $\varphi(x)$ are not always necessary in the equations.

As a example of dissipationless dynamical systems, we have integrated the equation for the Kepler motion

$$\frac{d^2\,x}{d\,t^2} = -\mu\frac{x}{(x^2+y^2)^{3/2}} , \qquad (1)$$

$$\frac{d^2\,y}{d\,t^2} = -\mu\frac{y}{(x^2+y^2)^{3/2}} , \qquad (2)$$

$$\mu = \pi^2/16 ,$$

$$x(0) = \frac{3}{4} , \quad \frac{d\,x(0)}{d\,t} = 0 ,$$

$$y(0) = 0 , \quad \frac{d\,y(0)}{d\,t} = \pi\sqrt{\frac{29}{192}} ,$$

where the constant μ and the initial conditions are chosen such that the analytic solution has period $T = 64$ and a relatively large value for the eccentricity. For the numerical computations by **HDMTDV**, we have used step size $h = 0.25(= T/256)$ and total time step number 10^6 $(0 \leq t \leq 5\times10^5)$. Plots of the error for energy and angular momentum are shown in Fig.1.

This new algorithm is described in detail in ref. [1].

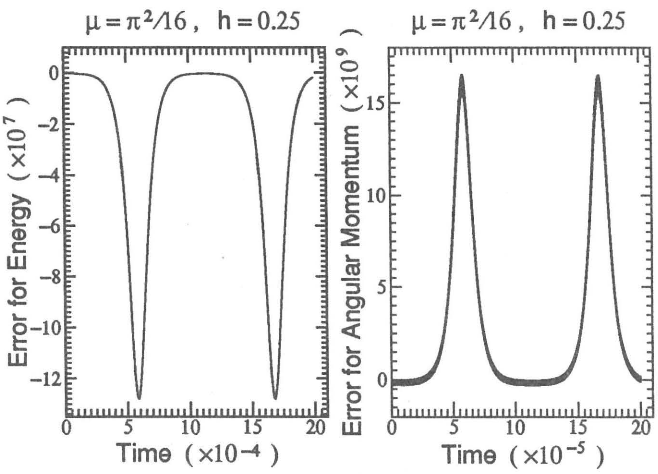

Fig.1 Plot of error for energy and angular momentum of the numerical solution of Kepler Motion. Because the period of the numerical solution is very close to the analytical value (= 64), the recursion time of the numerical solution is very long. This figure shows the secular error free computation characteristics of **HDMTDV**.

Reference

1) Watanabe,T. and Gnudi, G., NIFS-341 (1995)