§38. Unified Algorithm for Partial Differential Equations

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A new unified algorithm is proposed to solve partial differential equations which describe nonlinear boundary value problems, eigenvalue problems and time developing boundary value problems. The algorithm is composed of implicit difference scheme and multiple shooting scheme and is named as HIDM (Higher order Implicit Difference Method). A new prototype computer program HIDM2D for 2-dimensional partial differential equations is constructed and tested successfully to several problems, including Shroedinger equation

$$i\frac{\partial\psi}{\partial t} = -\frac{\partial^2\psi}{\partial x^2} + V(x)\psi\ , \quad (0 \le x \le 1) \tag{1}$$

Burger's equation

$$\frac{\partial\phi}{\partial t} + \phi\frac{\partial\phi}{\partial x} = \nu\frac{\partial^2\phi}{\partial x^2}, \quad (0 \le x \le 1) \tag{2}$$

and 2D eigenvalue problem

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = (V - \lambda)\psi \tag{3}$$

$$(\,-\infty \le x \le \infty\ , \quad 0 \le y \le 2\pi\,)$$

with potential

$$V(x,y) = 5\exp\left(-5\sin^2(x/2) - 2\sin^2(y/2)\right) \tag{4}$$

where λ represents eigenvalues. Fig.1 shows a band structure for this 2D eigenvalue problems.

Extension of the computer programs to 3 or more higher order dimension problems will be easy due to the direct product type difference scheme.

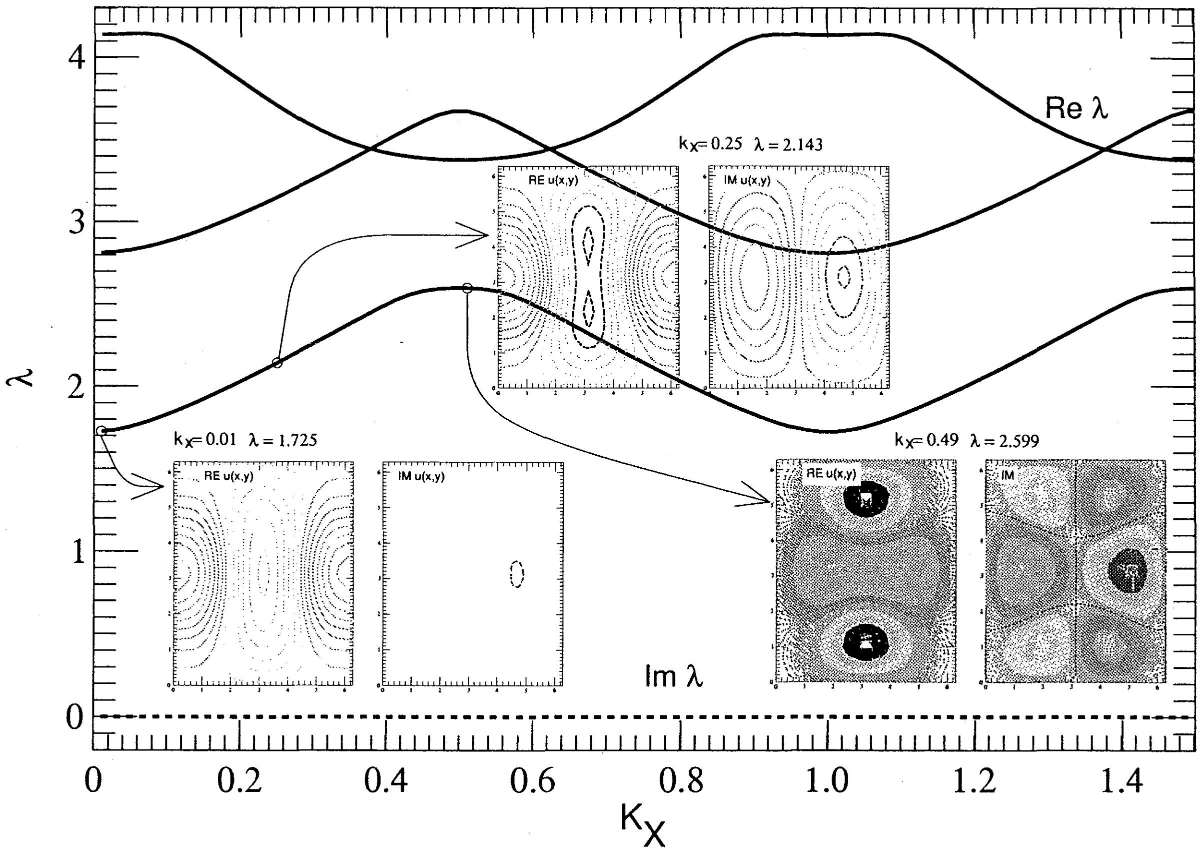

Fig.1 Numerical example of HIDM2D for 2D eigenvalue problem(3). The relation of the wavenumber k_x and energy eigenvalue λ is plotted. Typical structure of Bloch function $u(x,y)$ are also shown.

$$\psi(x,y) = \exp(ik_x x)u(x,y)$$