§33. An Automatic Roots Finding Method for Systems of Nonlinear Equations

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We have developed an algorithm for all solutions to nonlinear systems of equations

$$0 = h_i(x_1, x_2, \cdots, x_n),\ (i = 1, 2, , \cdots, n) \tag{1}$$

within a given compact domain $\boldsymbol{D} \in \boldsymbol{R}^n$ based on the numerical integration of the δ function (δ function method). The δ function method is proved to include the numerical algorithm for root finding problem based on the Cauchy integration theorem of an analytic function of 1 complex variable, and the bisection scheme of 1 real variable.

When we can guess approximate solutions, it is relatively easy to get highly accurate solutions for the nonlinear systems of equations (1) by the Newton iteration scheme. The delta function method is used to get all approximate solutions for eq.(1) with specified accuracy.

The delta function $\delta[h_1, h_2, \cdots, h_n]$ has the nature

$$\begin{aligned} I &\equiv \int_\Omega \delta[h_1, h_2, \cdots, h_n] dx_1 dx_2 \cdots dx_n \\ &= \begin{cases} 1 & \text{if } \Omega \text{ contains an solution for eq.(1).} \\ 0 & \text{if } \Omega \text{ contains no solution for eq.(1).} \end{cases} \end{aligned} \tag{2}$$

The δ function method is developed by the numerical integration of eq.(2) with following expressions for δ functions.

$$\delta(x_1, x_2) = \frac{1}{2\pi} \nabla^2 \ln r\ , \text{ case for } n = 2$$

$$\delta(x_1, \cdots, x_n) = -\frac{1}{(n-2)\Omega_n} \nabla^2 \frac{1}{r^{n-2}}\ , \text{case for } n \geq 3$$

where the distance r, the surface are Ω_n and differential operator ∇^2 are defined as follows.

$$\Omega_n = \begin{cases} \frac{(2\pi)^{n/2}}{(n-2)!!} & \text{if n is even.} \\ \frac{2(2\pi)^{(n-1)/2}}{(n-2)!!} & \text{if n is odd.} \end{cases} \tag{3}$$

$$\nabla^2 = \sum_{i=1}^{n} \left(\frac{\partial}{\partial x_i} \right)^2, \quad r = \left(\sum_{i=1}^{n} {x_i}^2 \right)^{1/2}. \tag{4}$$

The δ function method is successfully applied for various types of equations and various number of $n (\leq 7)$.

References

[1] 渡辺二太, 他, 京大数理解析研講究録 1040(1998)p1-8.

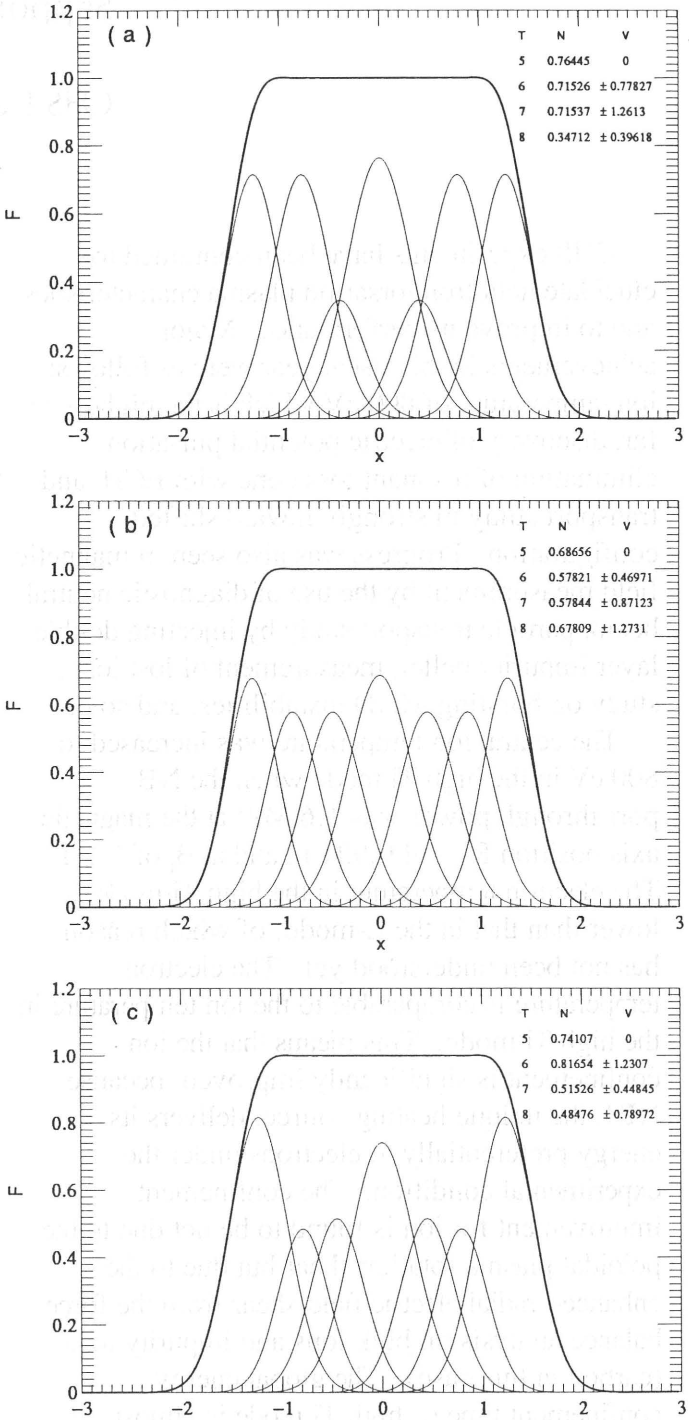

Fig.1 (a) The δ function method is applied to get flat top ($|x| < 1$) distribution function F(x) by superposition of 4 Gaussian distributions.

$$F(x) = \sum_{i=0,3} N_i \exp(-(x - V_i)^2 T_i)\,.$$

7 parameters N_i, V_i ($i = 0, 1, 2, 3$) are obtained numerically ($V_0 = 0$). The synthesized function $F(x)$ and Gaussian function elements are expressed by solid and thin lines. Total 7 sets of solutions are obtained numerically including solutions (a) ,(b) and (c).