J. Plasma Fusion Res. SERIES, Vol. 6 (2004) 144–147

Study of Finite-Orbit-Width Effect on Neoclassical Transport in Tokamak Core Region

SATAKE Shinsuke and OKAMOTO Masao
National Institute for Fusion Science, Toki 509-5292, Japan
(Received: 9 December 2003 / Accepted: 4 February 2004)

Abstract

Neoclassical transport simulation using the δf Monte-Carlo method is carried out to investigate the finite-orbit-width (FOW) effect on the transport near the magnetic axis. The time evolution of the radial electric field to maintain the ambipolarity of the flux is calculated simultaneously. It is found that, in the near-axis region, the ion heat flux decreases from the value predicted by the standard neoclassical theory both in the banana and plateau regimes. Though the radial transport shows a strong dependence on the FOW effect, the ambipolar electric field profile at the steady state is similar to that calculated in the small-orbit-width limit approximation.

Keywords:
neoclassical transport, finite-orbit-width effect, δf method, radial electric field, potato orbit

1. Introduction

Neoclassical transport theory has been studied in analytic approaches [1,2] in which small-orbit-width (SOW) approximation is used. In the analysis, the orbit width of particle drift motion is assumed to be negligibly small, though the finiteness of banana orbit does cause the neoclassical enhancement of particle and thermal transport. Recently, however, the finite-orbit-width (FOW) effect on the neoclassical transport has attracted attention. Especially in the region near the magnetic axis, where relatively wide-width potato particles dominate the transport process, transport theory in the SOW approximation breaks its assumption. There have been several attempts to investigate the transport phenomena in the near-axis region by using a direct simulation in Monte-Carlo method [3,4], or by an analytic way considering the existence of non-standard potato orbit around there [5,6]. Recently, we presented a new transport theory [7] to study this problem by using Lagrangian description of drift kinetic equation [8]. In this approach, in which the finiteness of real orbit widths is appropriately considered, it is shown that the ion heat conductivity χ_i in the near-axis region becomes much lower than that is predicted from the standard theory. In our previous study, the radial electric field has been neglected. However, it is shown [9] that the radial electric field is important in neoclassical transport with the FOW effect. The intrinsic ambipolarity is not valid if the FOW effect is included, and the radial electric field develops so that it retains $\Gamma_i(E_r) = \Gamma_e(E_r)$. And once a sheared E_r profile is established, it modifies banana width known as the orbit-squeezing effect, which in turn affects the radial fluxes. In order to study neoclassical transport with consistent E_r, one has to solve the time development of it according to the equation below

$$\left(1+\frac{c^2}{v_A^2}\right)\varepsilon_0\frac{\partial E_r(r,t)}{\partial t} = -Z_i e\langle \Gamma_i^{neo}(r,E_r,t)\rangle, \qquad (1)$$

where $\langle\cdots\rangle$ means a flux-surface average, and the electron flux is neglected. It is important to investigate the formation of the E_r profile to give a neoclassical steady state, but a consistent treatment of this problem is difficult in an analytic way.

In this paper, we investigate the FOW effect on neoclassical transport with radial electric field by using a Monte-Carlo simulation called the δf method. In Sec. 2, the basic formulation of the δf method is reviewed. In Sec. 3, simulation results are shown in the banana and plateau regimes. The SOW-limit model calculation using the δf method, which corresponds to conventional theory, is also shown. The summary of this paper is given in Sec. 4.

2. Formulation of δf method

The time development of gyro-averaged distribution function of a species a is described by the drift-kinetic equation as follows

$$\frac{\partial}{\partial t}f_a(\boldsymbol{x},\mathcal{K},\mu,t)+(\boldsymbol{v}_\parallel+\boldsymbol{v}_d)\cdot\nabla f_a+\dot{\mathcal{K}}\frac{\partial f_a}{\partial \mathcal{K}} = \sum_b C(f_a,f_b), \qquad (2)$$

Corresponding author's e-mail: satake@nifs.ac.jp

where $K = mv^2/2$, $\mu = mv_\perp^2/2B$, and $C(f_a, f_b)$ is the Fokker-Plank collision term. We consider here only the ion transport and the ion-electron collision is omitted. The δf method is based on the separation of the distribution function in the form $f = f_0 + \delta f$. Then eq. (2) is expanded order by order by assuming $\delta f/f_0 \sim v_d/v_\parallel \sim \delta \ll 1$. The lowest order is

$$\frac{\partial f_0}{\partial t} + \boldsymbol{v}_\parallel \cdot \nabla f_0 = C(f_0, f_0). \tag{3}$$

The time-independent solution of eq. (3) is given by a local Maxwellian

$$f_0 = f_M = n\left(\frac{m}{2\pi T}\right)^{3/2} \exp\left(-\frac{K}{T}\right) \tag{4}$$

with n and T being a flux-surface function. The next order equation becomes

$$\left(\frac{\partial}{\partial t} + (\boldsymbol{v}_\parallel + \boldsymbol{v}_d)\cdot\nabla + e\boldsymbol{v}_d \cdot \boldsymbol{E}_r \frac{\partial}{\partial K} - C(\,, f_M)\right)\delta f$$
$$= -\boldsymbol{v}_d \cdot \left(\nabla - \frac{e\boldsymbol{E}_r}{T}\right) f_M + C(f_M, \delta f), \tag{5}$$

where $\dot{K} = e\boldsymbol{v}\cdot\boldsymbol{E}$ is used, and only the radial electric field is considered here. In the δf method, only the δf part is solved according to eq. (5). The advantage of this method is the reduction of statistical noise, which is about $|\delta f/f_0|^2$ times lower than that in a full-f calculation.

The linearized collision term is treated in a Monte-Carlo way. The test-particle collision operator $C_{TP}(\delta f) = C(\delta f, f_M)$ is implemented by random kicks in the velocity space [10]. The field-particle operator $C(f_M, \delta f)$ is approximated by a linear operator $\mathcal{P} f_M$ which satisfies conservation lows of collision term

$$\int d^3v \mathcal{M}(C_{TP} + \mathcal{P} f_M) = 0 \tag{6}$$

for $\mathcal{M} = 1$, $v_\parallel$, and K. In the simulation, δf is expressed by simulation markers of which distribution is $g = \sum_i \delta(\boldsymbol{x} - \boldsymbol{x}_i)\delta(\boldsymbol{v} - \boldsymbol{v}_i)$, and two weights w and p for each marker which satisfy the relations $wg = \delta f$, $pg = f_M$. Each marker moves so as to satisfy $Dg/Dt = 0$, where D/Dt is the lhs of eq. (5). Using these relations in eq. (5) yields the evolution of two weights as follows

$$\dot{w} = \frac{p}{f_M}\left[-\boldsymbol{v}_d \cdot \left(\nabla - \frac{e\boldsymbol{E}_r}{T}\right) + \mathcal{P}\right] f_M, \tag{7}$$

$$\dot{p} = \frac{p}{f_M} \boldsymbol{v}_d \cdot \left(\nabla - \frac{e\boldsymbol{E}_r}{T}\right) f_M. \tag{8}$$

The self-consistent radial electric field is obtained from eq. (1). The neoclassical particle and heat fluxes are evaluated as

$$\Gamma^r_{neo} = \left\langle \int d^3v\, \boldsymbol{v}_d \cdot \hat{r} w g \right\rangle, \tag{9}$$

$$\boldsymbol{q}^r_{neo} = \left\langle \int d^3v\, \boldsymbol{v}_d \cdot \hat{r} K w g \right\rangle, \tag{10}$$

where $\hat{r}$ is unit vector normal to the flux surface. Note here that the FOW effect in this formulation arises from the "$\boldsymbol{v}_d\cdot$" terms in the lhs of eq. (5). Neglecting these terms results in the reduction to the SOW-limit neoclassical transport [1,2] where these terms are treated as higher-order ones and thus excluded from eq. (5). In the next section, we compare the SOW-limit calculation with the FOW case to investigate how the finiteness of particle orbit affects radial transport. In the SOW-limit case, only the parallel motion of marker is solved, and eq. (8) is replaced by $\dot{p} = 0$. The equation for $\dot{w}$ is unchanged since eq. (7) arises from the rhs of eq. (5).

3. Simulation results

Simulations are carried out in two collisional regimes; in banana and plateau regimes. Estimation of collisionality in the near-magnetic axis region is given as follows [4,7]

$$\nu_*(r = r_p) \ll 1 \quad \cdots \text{banana}$$
$$1 \ll \nu_*(r = r_p) \ll (R_0/r_p)^{3/2} \quad \cdots \text{plateau}$$

where $r_p = 2(2q^2\rho^2R_0)^{1/3}$ is the fattest potato width passing through the magnetic axis, and $\nu_* = \nu_c^{eff}/\omega_b$ is the fraction of the effective collision frequency to the bounce frequency [1]. We use a simple configuration of circular cross-section with $R_0 = 4$ m, $a = 1$ m, and $q = 2$. Profiles of n and T have an exponential slope. The collisionality is controlled by changing n_0, T_0, and B_0 so that the potato width r_p is unchanged (0.25 m) in both cases. Initial E_r-profile is given from Hinton-Hazeltine's relation [1] between $u_\parallel$ and E_r in a steady state, where we set $u_\parallel = 0$ initially. Both in the two collisionality, we carry out a normal δf simulation including the FOW effect and the SOW-limit case in which the radial drift motion of markers are neglected.

3.1 Collisionless case

In this case, we choose $\nu_*(r_p) \simeq 0.12$. Since $\nu_* \ll 1$, a potato particles can complete its orbit before it is scattered by collisions. Then the finiteness of potato orbit is expected to affect neoclassical transport around the magnetic axis. The numerical result of the ion heat flux is shown in Fig. 1.

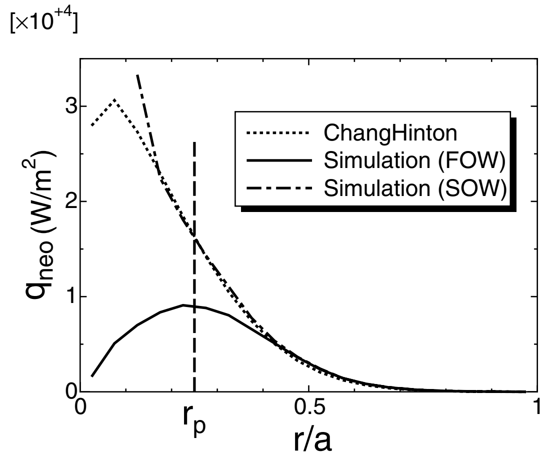

Fig. 1 Ion heat flux in $\nu_* \simeq 0.12$ case.

Around the magnetic axis, one can see that $\boldsymbol{q}_{neo}$ drops from the expectation by the conventional neoclassical theory [11], in which the existence of potato particles have been neglected. The FOW effect on $\boldsymbol{q}_{neo}$ appears in the region $r < 1.5r_p$ where potato particles dominate transport process. The same tendency has also been seen in our analytic theory considering the FOW effect [7], in which the drop begins at $r \sim r_p$. The result of SOW-limit calculation agrees well with the conventional theory, though we cannot have a reliable value at $r < 0.1$ because of an unexpected accumulation of numerical noise.

In Fig. 2, the E_r and $\langle u_\parallel \rangle$ profiles in the FOW-case are shown. Radial transport of toroidal angular momentum develops the $u_\parallel$ profile in time, but its value remains negligible compared to the thermal velocity. Though the radial transport is significantly modified, the steady-state E_r profile is almost the same as its initial value which is given by a conventional theory. It is of interest that the steady-state E_r profile in the SOW-limit case is similar to the FOW case. Then it seems that the neoclassical steady state is affected little from the non-locality of the radial transport. Precise investigation will be done in the future by introducing a non-zero initial $u_\parallel$ profile.

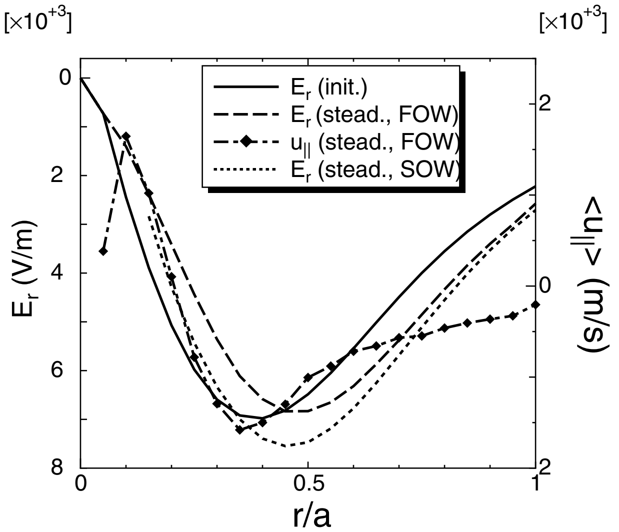

Fig. 2 E_r and $\langle u_\parallel \rangle$ profiles ($\nu_* \simeq 0.12$).

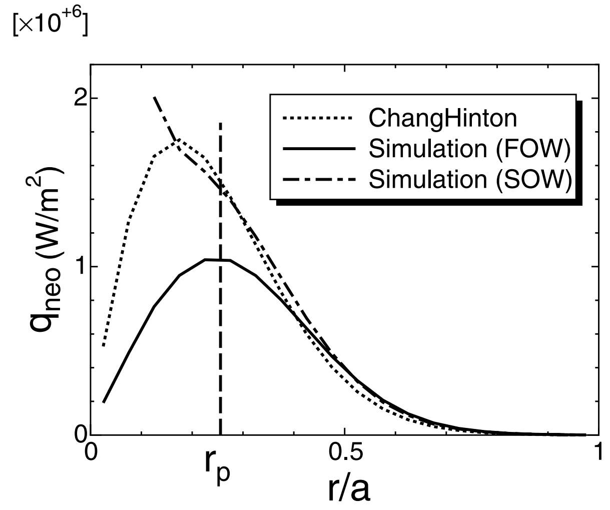

Fig. 3 Ion heat flux in $\nu_* \simeq 10$ case.

3.2 Collisional case

This case we choose $\nu_* (r_p) \simeq 10$. In this situation, potato particles are perturbed by collisions before they complete their full orbit. Moreover, the main contribution to the radial transport comes from resonant passing particles. Then it has been thought that the plateau transport in the near-axis region is not affected by potato particles [12]. However, as can be seen in Fig. 3, the ion heat flux decreases at the near-axis region like as in the banana regime case. This is contrastive to the previous δf calculation [4] without E_r, which shows a good agreement of the resulting χ_i with the theoretical value at $\nu_* (r_p) \geq 10$. Since the convective heat flux $q_{conv.} = 1.5\Gamma_i T_i$ remains substantially if E_r is not solved, the difference is partly caused by the ambipolar E_r in our simulation. It is also to be noted that the resonant particles, which are characterized by a small $|v_\parallel / v|$, have a wide orbit width as potato in the near-axis region [13]. Therefore, the transport in the plateau regime should not be simply considered in an analogy with the transport theory valid in the region away from the axis. We are planning a more precise analysis on the plateau regime transport in the near-axis region using δf simulation.

4. Summary

Neoclassical transport around the magnetic axis has been investigated by δf transport simulation with radial electric field. Both in the banana and plateau regimes, the ion heat flux is found to decrease from the theoretical value which neglects potato orbits. We have demonstrated that the δf method can reproduce the conventional neoclassical theory in the SOW limit by omitting the drift motion of simulation markers. Though the radial transport is modified largely by the existence of potato particles, it is found that the steady-state E_r profiles are similar between the FOW- and SOW-limit simulation results. This work is supported by JSPS Research Fellowships for Young Scientists, No. 10177.

References

[1] F.L. Hinton and R.D. Hazeltine, Rev. Mod. Phys. **48**, 239 (1976).
[2] S.P. Hirshman and D.J. Sigmar, Nucl. Fusion **21**, 1079 (1981).
[3] Z. Lin *et al.*, Phys. Rev. Lett. **78**, 456 (1997).
[4] A. Bergmann *et al.*, Phys. Plasmas **8**, 5192 (2001).
[5] K.C. Shaing *et al.*, Phys. Plasmas **4**, 771 (1997).
[6] P. Helander, Phys. Plasmas **7**, 2878 (2000).
[7] S. Satake, M. Okamoto and H. Sugama, Phys. Plasmas **9**, 3946 (2002).
[8] I.B. Bernstein and K. Molvig, Phys. Fluids **26**, 1488 (1983).
[9] W.X. Wang *et al.*, Phys. Rev. Lett. **87**, 055002 (2002).

[10] X.Q. Xu and M.N. Rosenbluth, Phys. Fluids B **3**, 627 (1991).
[11] C.S. Chang and F.L. Hinton, Phys. Fluids **25**, 1493 (1982).
[12] P. Helander, Phys. Plasmas **7**, 2878 (2000).
[13] S. Satake, *et al.*, J. Plasma Fusion Res. **78**, 573 (2001).